Modelling and control summaries

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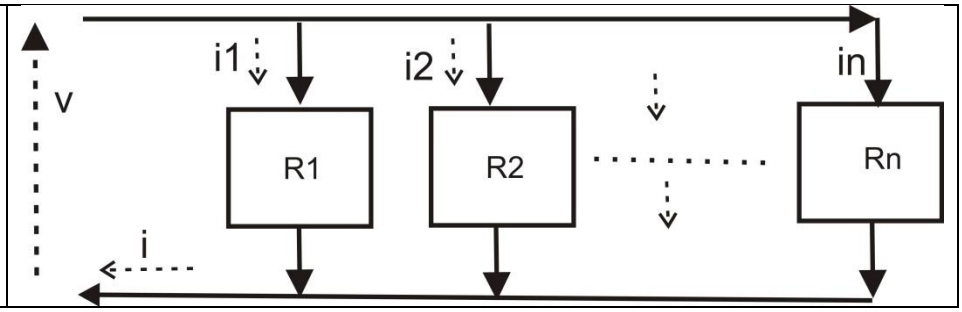
Modelling principles and analogies 4:

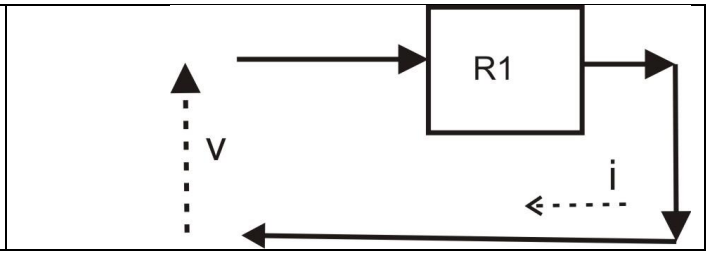
Modelling resistors in parallel

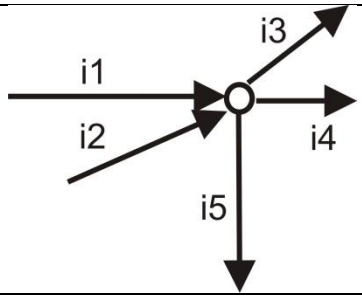
Consider an electrical circuit with supplied voltage *v(t)*, a set of resistors arranged in parallel (such as in figure here) and an overall current flow *i(t)*.

What are the relationships between the currents, the applied voltage and the resistances?	

These notes will give a brief overview of the foundational assumptions required to answer this question.

Model for a single resistor

The current flow is proportional to the voltage. Specifically, the dependence can be expressed as: $$v = iR_1$$	

Kirchhoff's Current Law All the currents acting at a single point must add to zero. $$i_1 + i_2 + \cdots + i_n = 0$$	

Assumption: Electrical components in parallel share the same voltage.

Combining the rule for a single resistor with Kirchhoff's current law and the assumption, one can now derive a model for many resistors in parallel.

Model for a two resistors in parallel

Modelling requires a statement of each observation above and a simplification of the resulting equations.

1. The current across resistor R1

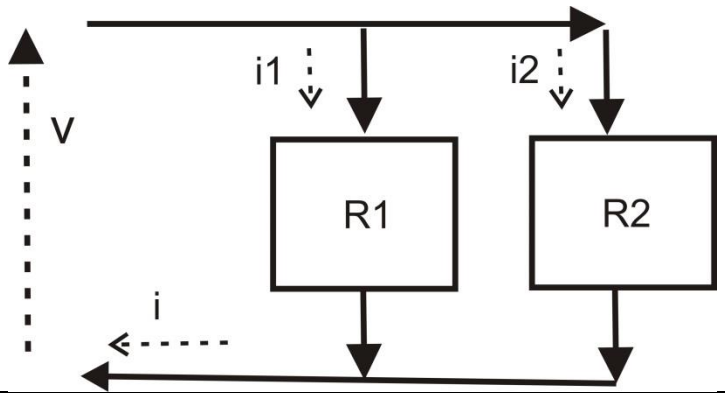
$$v = i_1 R_1$$

2. The current across resistor R2

$$v = i_2 R_2$$

3. Kirchhoff's current law:

$$i_1 + i_2 = i$$



Combining these three equations gives:

$$\frac{v}{R_1} + \frac{v}{R_2} = i_1 + i_2 = i \quad \Rightarrow \quad v = i\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}\right); \quad R_T = \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}\right)$$

Model for a many resistors in parallel

Modelling requires a statement of each observation above and a simplification of the resulting equations.

1. The current across resistor R1

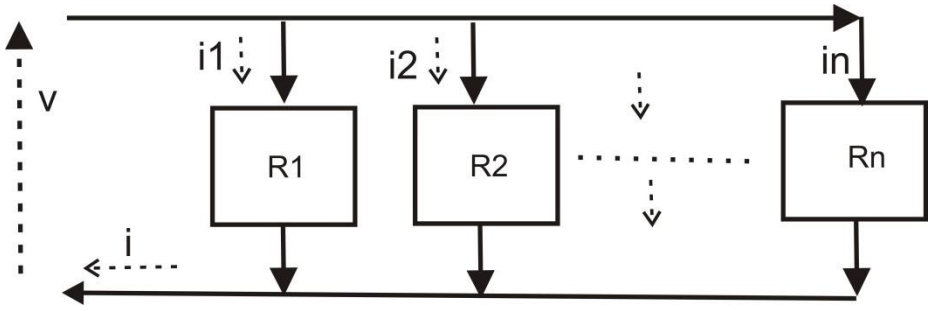
$$v = i_1 R_1$$

2. The current across resistor Rj

$$v = i_j R_j$$

3. Kirchhoff's current law:

$$i_1 + \cdots + i_n = i$$

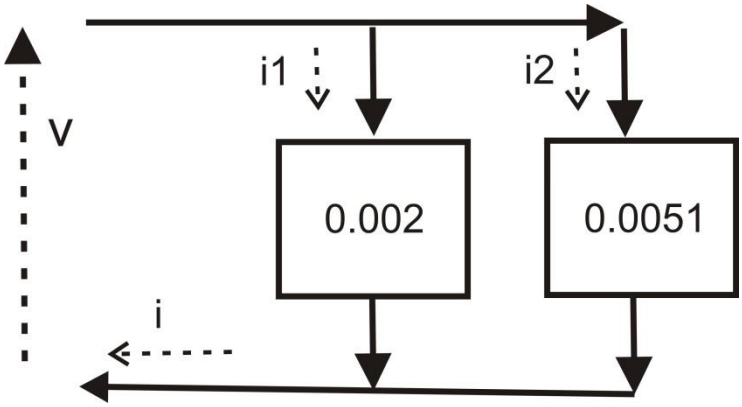


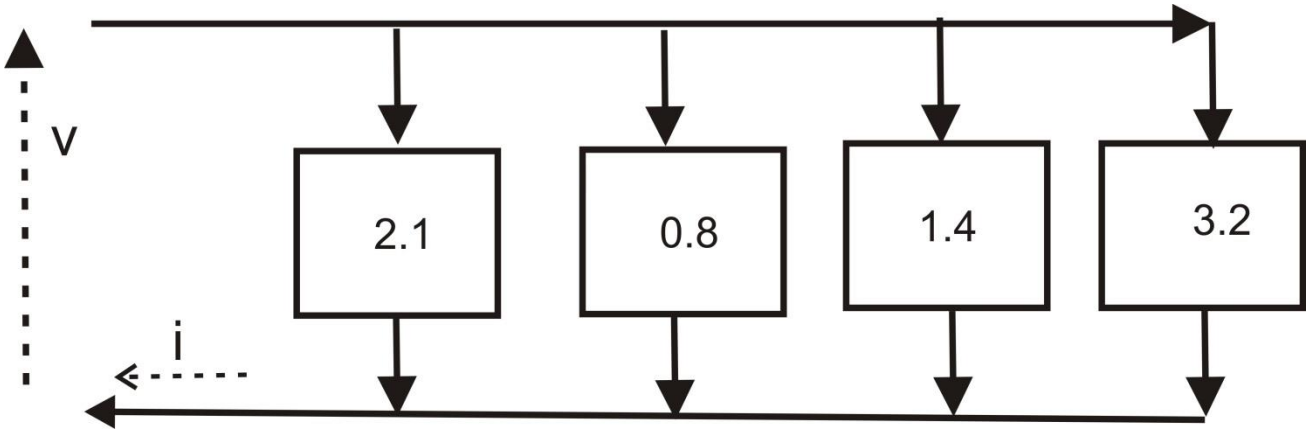
Combining these equations gives:

$$\frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_n} = i_1 + i_2 + \cdots + i_n = i \quad \Rightarrow \quad v = i\underbrace{\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}\right)}_{R_t}$$

Tutorial questions

Find the overall circuit resistance for the following circuits. (Any numbers given represent the resistance in ohms).





Which has greatest resistance from: (i) a circuit with two equal resistors in series and (ii) a circuit with 5 equal resistors in parallel?

- In general, does adding a resistor in parallel reduce in increase overall resistance?
- In general, does adding a resistor in series reduce or increase overall resistance?

ANSWERS: (i) 0.0014ohms (ii) 0.3632ohms;