Modelling and control summaries

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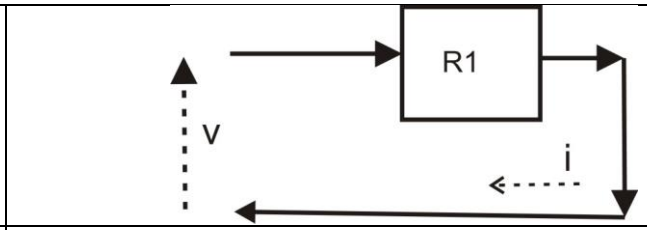
Modelling principles and analogies 5:

Pipes in series and parallel

Flow through pipes is exactly analogous to current flow in systems of electrical resistors (assuming steady-state flows and local linearity). Non-linear effects over large pressure ranges and transient effects are outside the scope of this note.

Model for a single resistor and analogy to model for a single pipe

The current flow is proportional to the voltage. Specifically, the dependence can be expressed as: $$v = iR_1$$	
The fluid flow is proportional to the pressure difference, and hence: $$P1 - P2 = FR_1$$	

The analogies are obvious:

Pressure difference P1-P2 is analogous to voltage v.	**Fluid flow F is analogous to current flow i.**	**Electrical resistance is analogous to pipe flow resistance.**

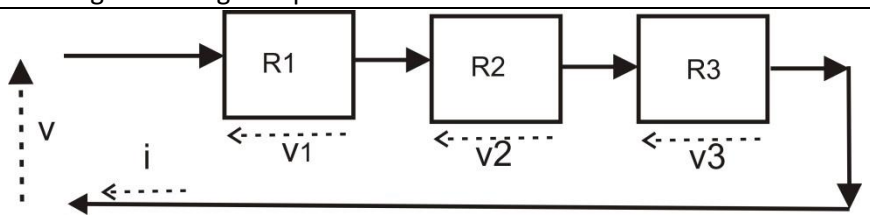
MANY pipes in series are analogous to many electrical resistors in series

Many pipes in series can be modelled by considering the pressure drop across each pipe in turn given equations as follows:

$$\left.\begin{aligned} P1 - P3 &= FR_1 \\ P3 - P4 &= FR_2 \\ P4 - P2 &= FR_3 \end{aligned}\right\} \Rightarrow \quad P1 - P2 = F(R_1 + R_2 + R_3);$$

Analogously, resistors in series considering the voltage drop across each resistor in turn:

$$\left.\begin{aligned} v1 &= iR_1 \\ v2 &= iR_2 \\ v3 &= iR_3 \end{aligned}\right\} \Rightarrow$$



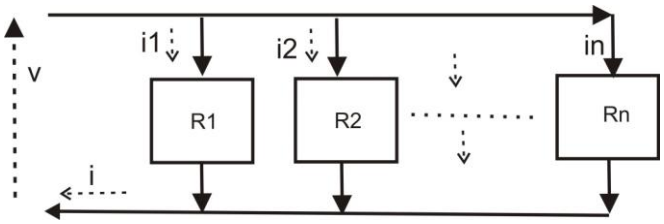
$$v = i(R_1 + R_2 + R_3);$$

MANY pipes in parallel are analogous to many electrical resistors in parallel

Again, using the analogies between the underlying models for single pipes and single resistors, it is clear that one gets analogous models for many pipes/resistors in parallel. Just as currents add for resistors in parallel, then flows must add for pipes in parallel – the more pipes you have in parallel the greater the overall flow and thus the smaller the implied resistance to flow.

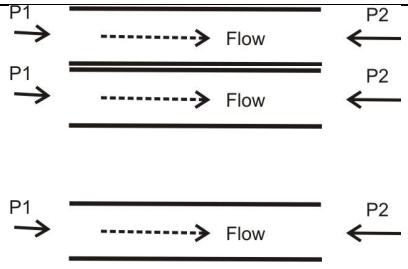
$$\frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_n} = i_1 + i_2 + \cdots + i_n = i$$

$$\Rightarrow \quad v = i \underbrace{\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \right)}_{R_t}$$



$$\frac{\delta P}{R_1} + \frac{\delta P}{R_2} + \cdots + \frac{\delta P}{R_n} = F_1 + F_2 + \cdots + F_n = F$$

$$\Rightarrow \quad \delta P = F \underbrace{\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \right)}_{R_t}$$



$$\delta P = P1 - P2$$

REMARK: It is perhaps obvious that pipes with larger cross-sectional areas have smaller resistance to flow as this is equivalent to joining in parallel many smaller pipes!
Similarly, longer pipes have more resistance to flow than shorter pipes.