Modelling and control summaries

by Anthony Rossiter

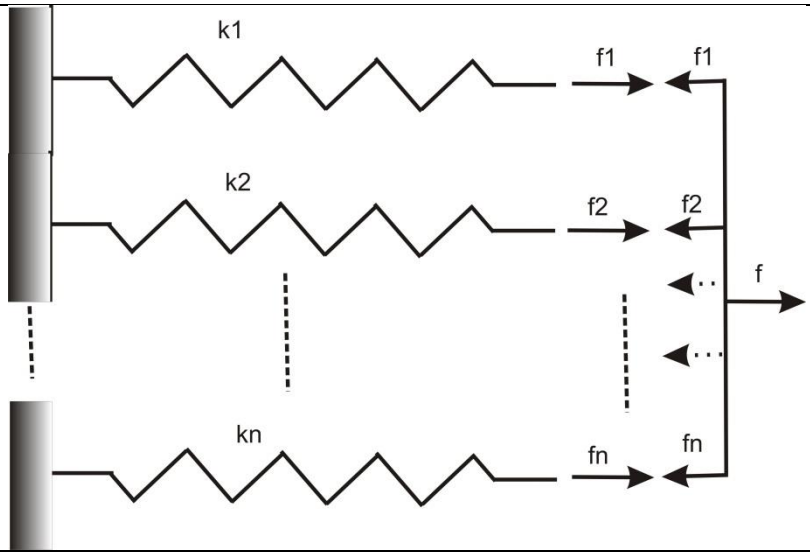
Modelling principles and analogies 6:

Modelling springs in parallel

Consider a set of springs of identical length which are fixed at one end and then arranged in parallel so that the same force is extending all of them simultaneously and they all therefore also have the same deflection.

What is the relationship between the applied force, the displacement and the spring stiffnesses?

These notes will give a brief overview of the foundational assumptions required to answer this question.



Model for a single spring

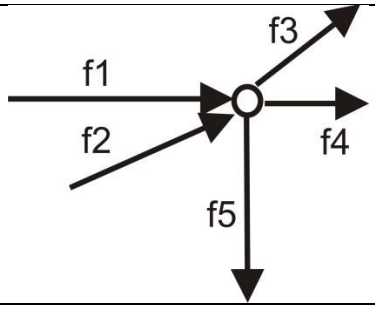
The displacement of a linear spring is proportional to the applied force. Specifically, for spring stiffness kN/m, the dependence can be expressed as:

$$f = kx$$

Equivalent of Kirchhoff's Voltage Law for mechanical components in parallel

All the forces (treat as vectors) acting at a single point must sum to zero.

$$f_1 + f_2 + f_3 + f_4 + f_5 = 0$$



Assumption here: Mechanical components in parallel are sharing the same supplied force and all have the same displacement (obviously this may not always be the case and users must be careful with their own scenarios).

Combining the rule for a single spring with the force balance equation Kirchhoff's voltage law and the assumption above, one can now derive a model for many springs in parallel.

Model for a two springs in parallel

Modelling simply requires a statement of each observation above and then a simplification of the resulting equations. Assume the springs have stiffness k_1 N/m and k_2 N/m.

1. The displacement of spring 1

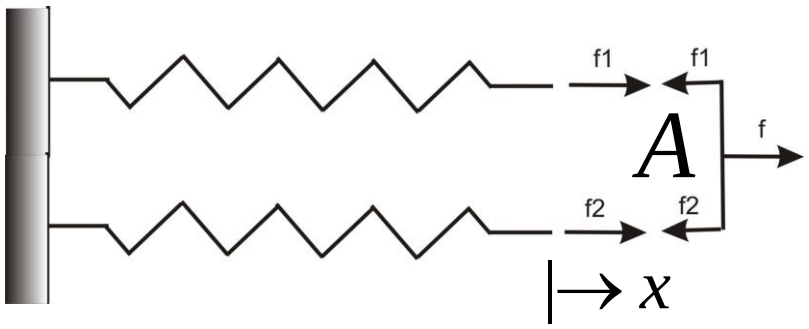
$$f_1 = k_1 x$$

2. The displacement of spring 2

$$f_2 = k_2 x$$

3. Force balance at 'A':

$$f_1 + f_2 = f$$



Combining these three equations gives:

$$k_1 x + k_2 x = f \quad \Rightarrow \quad f = (k_1 + k_2)x$$

Model for a n resistors in series

Modelling requires a statement of each observation above and a simplification of the resulting equations.

1. The displacement of spring 1:

$$f_1 = k_1 x$$

2. The displacement of spring 2:

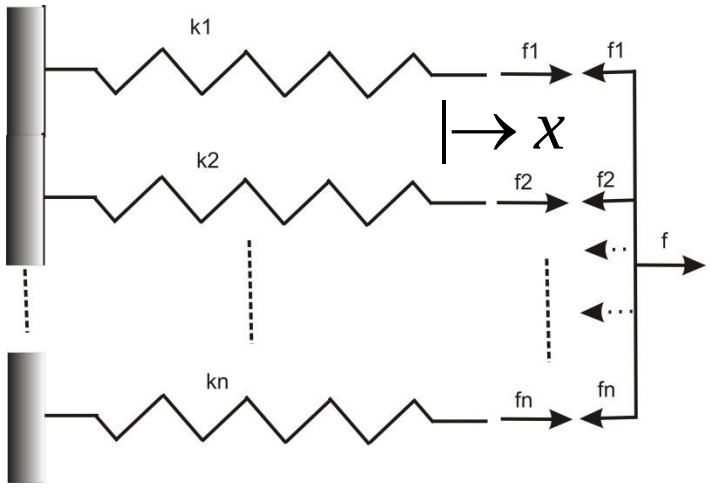
$$f_2 = k_2 x$$

3. The displacement of spring i:

$$f_i = k_i x$$

4. Force balance:

$$f_1 + f_2 + \cdots + f_n = f$$

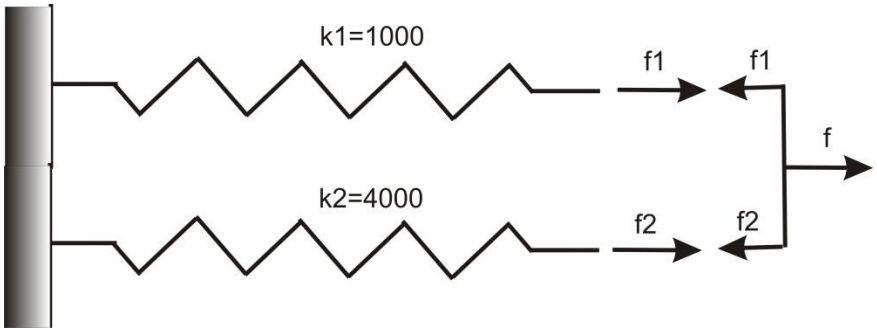


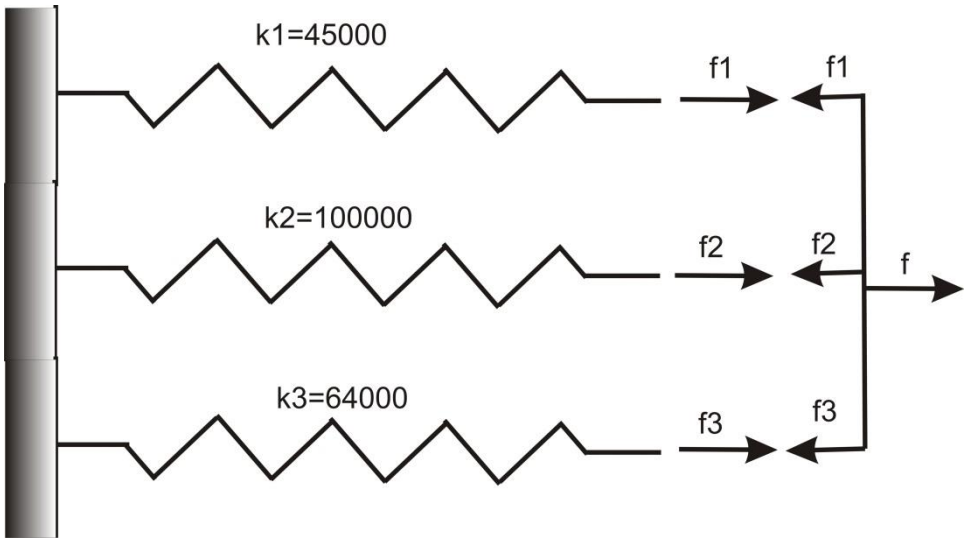
Combining these equations gives:

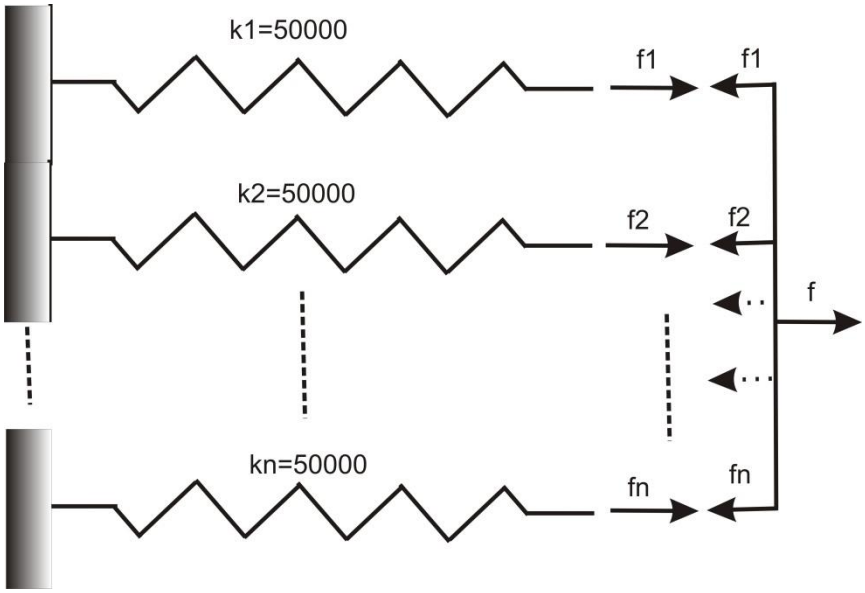
$$k_1 x + k_2 x + \cdots + k_n x = f \quad \Rightarrow \quad f = (\underbrace{k_1 + k_2 + \cdots + k_n}_{k_t})x$$

Tutorial questions

Find the overall stiffnesses for the following arrangements of springs. Any numbers given represent the individual stiffness in N/m.







ANSWERS: (i) 5000 kN/m (ii) 209000 kN/m; (iii) 50000n kN/m