Modelling and control summaries

by Anthony Rossiter

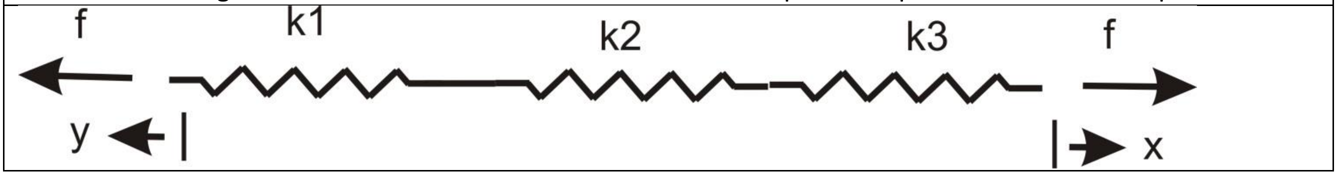
Modelling principles and analogies 7:

Modelling springs in series

Consider a set of springs of identical length which are arranged in series so that the same force is passing through all of them simultaneously. What is the overall stiffness and extension?

What is the relationship between the applied force f, the overall extension (x-y) and the spring stiffnesses?

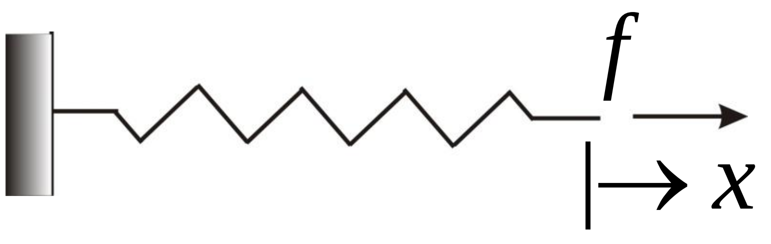
These notes will give a brief overview of the foundational assumptions required to answer this question.



Model for a single spring

The extension of a linear spring is proportional to the applied force. Specifically, for spring stiffness kN/m, the dependence can be expressed as:

$$f = ke$$



Equivalent of Kirchhoff's current Law for mechanical components in parallel

The same force must pass through all the mechanical components arranged in series (assuming no acceleration of masses).

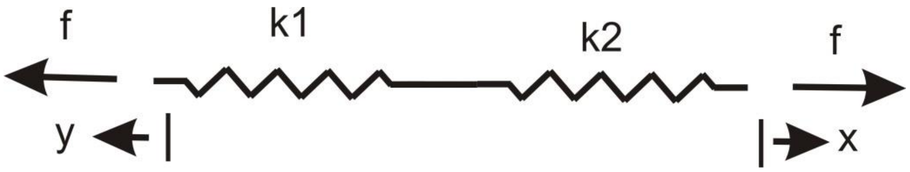
The overall extension is the sum of all the extensions. Overall extension is also difference between movement of one end 'x' and movement of the other end 'y'.

$$e_1 + e_2 + \cdots + e_n = e = x - y$$

Combining the rule for a single spring with the observation one can now derive a model for many springs in series.

Model for two springs in series

Modelling simply requires a statement of each observation above and then a simplification of the resulting equations. Assume the springs have stiffness k_1 N/m and k_2 N/m.

1. The extension of spring 1 $f = k_1 e_1$ 2. The extension of spring 2 $f = k_2 e_2$ 3. Overall extension: $e_1 + e_2 = x - y$ Combining these three equations gives:	 $\frac{f}{k_1} + \frac{f}{k_2} = x - y \quad \Rightarrow \quad f = \left(\frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} \right)(x - y)$

Model for n springs in series

Modelling requires a statement of each observation above and a simplification of the resulting equations.

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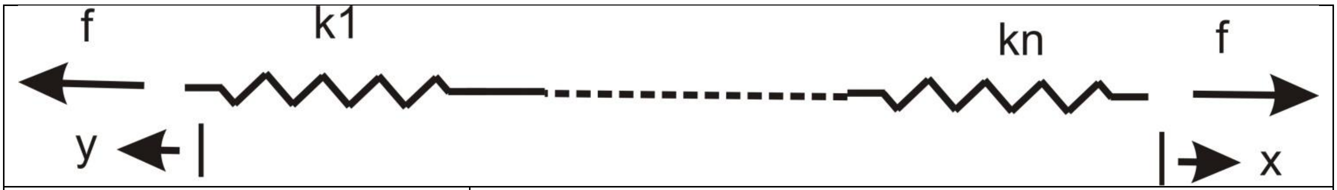
<tr><td colspan="2">
</td></tr>
<tr><td>1. The extension of spring 1:
$f_1 = k_1 x$
2. The extension of spring i:
$f_2 = k_2 x$
3. Overall extension:
$e_1 + e_2 + \cdots + e_n = x - y$</td><td>Combining these equations gives:
$\frac{f}{k_1} + \cdots + \frac{f}{k_n} = x - y \quad \Rightarrow \quad f = \underbrace{\left(\frac{1}{\frac{1}{k_1} + \cdots + \frac{1}{k_n}} \right)}_{k_t}(x - y)$</td></tr>
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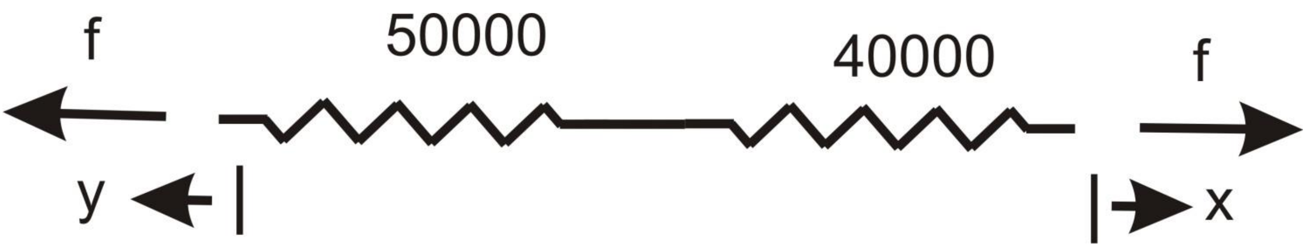
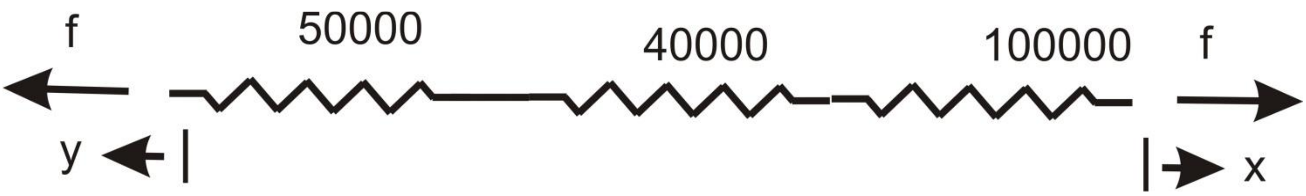
Tutorial questions

Find the overall stiffnesses for the following arrangements of springs. Any numbers given represent the individual stiffness in N/m.





Which has greatest stiffness from: (i) a system with 4 equal springs in series and (ii) a system with 2 equal springs in parallel?

- In general, does adding a spring in parallel reduce or increase overall stiffness?
- In general, does adding a spring in series reduce or increase overall stiffness?

ANSWERS: (i) 22222 kN/m (ii) 18182 kN/m;