Modelling and control summaries

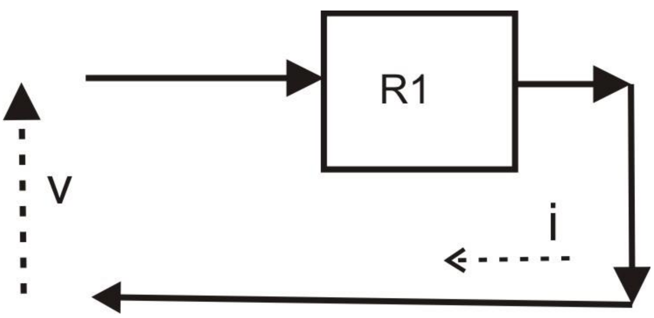
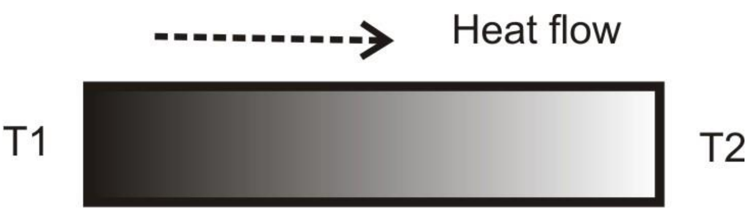
by Anthony Rossiter

The University Of Sheffield.

Modelling principles and analogies 8:

Summaries

A number of apparently different systems have analogous models. There is a simple proportional relationship between the driving input (voltage difference, pressure difference, temperature difference, force) and the resulting movement (current flow, fluid flow, heat flow, extension).

	Current through a resistor has is proportional to the voltage difference across the resistor. $(v1-v2)=Ri$
	Flow through a pipe is proportional to the pressure difference across the pipe. $(P1-P2)=K_P F$
	Heat flow through a conductor is proportional to the temperature difference across the ends. $(T1-T2)=K_H W$
	Extension of a spring is proportional to the applied force. $f=ke$

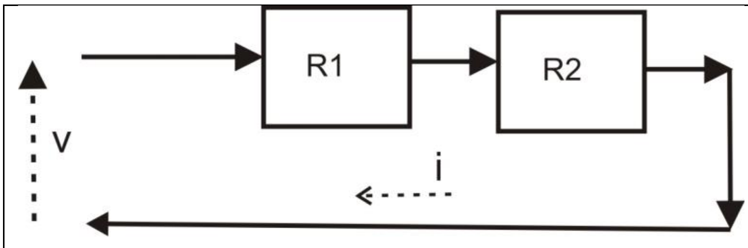
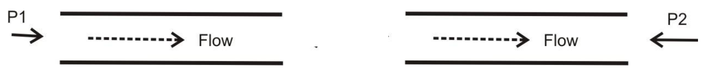
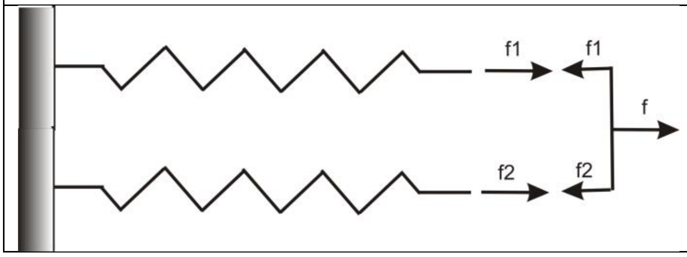
OBSERVATIONS 1

When arranged in multiples, analogous models continue to be apparent, but the analogies may be between parallel and series arrangements. If voltage (or Pressure/Temperature difference) is the distributed across many components (series arrangement) then:

$$\delta v = i\sum_j R_j; \quad \delta P = F\sum_j K_j; \quad \delta T = W\sum_j K_j;$$

A similar equation results if a force is distributed across parallel components such as springs.

$$f = x\sum_j k_j;$$

	Resistors in series, the overall resistance ADDS! $v1 - v2 = (R1 + R2 + \cdots)i$
	Pipes in series, the overall resistance to flow ADDS! $P1 - P2 = (K_{P1} + K_{P2} + \cdots)F$
	Conductors in series, the overall resistance to heat flow ADDS! $T1 - T2 = (K_{H1} + K_{H2} + \cdots)W$
	Springs in parallel, the overall stiffness (resistance to movement) ADDS! $f = (k1 + k2)e$

REMARK: Although not discussed here, readers may like to investigate other components and scenarios which have analogous equations. Examples could include capacitors, inductors, dampers, tanks, … .

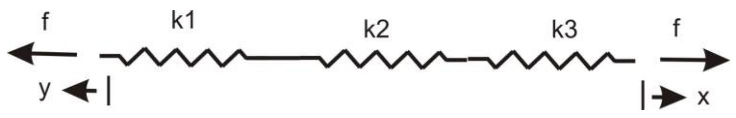
OBSERVATIONS 2:

Resistors (pipes and conductors) in parallel share the same voltage (pressure/temperature) and the current (flow) is divided among them. **Total flow is sum of flows across each component.**

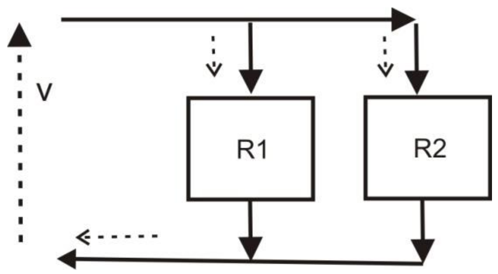
$$\left\{ i = \sum_j i_j; \quad i_j = \frac{\delta v}{R_j}; \right\} \quad \left\{ F = \sum_j F_j; \quad F_j = \frac{\delta P}{K_j}; \right\} \quad \left\{ W = \sum_j W_j; \quad W_j = \frac{\delta T}{K_j} \right\}$$

Springs in series share the same force and the displacement is divided among them. Total extension is sum of extensions across each component.

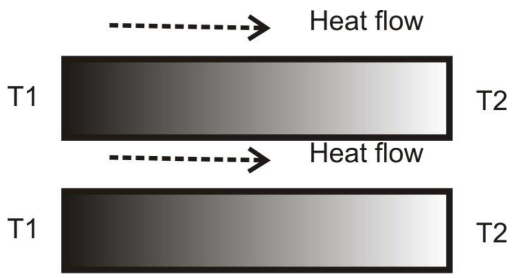
$$\left\{ e = \sum_j e_j; \quad e_j = \frac{f}{k_j}; \right\}$$

<table>
<tr><td>


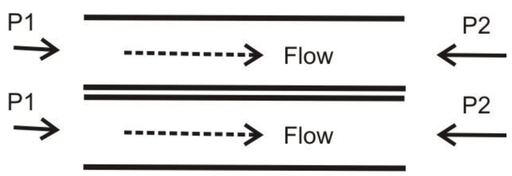
</td><td>Springs in series

$$\left\{ x - y = \sum_j e_j; \quad e_j = \frac{f}{k_j}; \right\}$$
</td></tr>
<tr><td>


</td><td>Resistors in parallel.

$$\left\{ i = \sum_j i_j; \quad i_j = \frac{\delta v}{R_j}; \right\}$$
</td></tr>
<tr><td>


</td><td>Heat conductors in parallel

$$\left\{ W = \sum_j W_j; \quad W_j = \frac{\delta T}{K_j} \right\}$$
</td></tr>
<tr><td>


</td><td>Pipes in parallel

$$\left\{ F = \sum_j F_j; \quad F_j = \frac{\delta P}{K_j}; \right\}$$
</td></tr>
</table>