

Modelling and control summaries



by Anthony Rossiter

Modelling principles and analogies 8:

Summaries

A number of apparently different systems have analogous models. There is a simple proportional relationship between the driving input (voltage difference, pressure difference, temperature difference, force) and the resulting movement (current flow, fluid flow, heat flow, extension).

<p>A circuit diagram showing a rectangular resistor labeled 'R1'. A solid arrow points from left to right into the resistor. A solid arrow points from right to left out of the resistor. A dashed arrow labeled 'i' points from right to left below the resistor. On the left side, a vertical dashed arrow labeled 'v' points upwards, representing the voltage difference across the resistor.</p>	<p>Current through a resistor has is proportional to the voltage difference across the resistor.</p> $(v1 - v2) = Ri$
<p>A diagram of a horizontal pipe. On the left end, a solid arrow labeled 'P1' points into the pipe. On the right end, a solid arrow labeled 'P2' points into the pipe. A dashed arrow labeled 'Flow' points from left to right inside the pipe.</p>	<p>Flow through a pipe is proportional to the pressure difference across the pipe.</p> $(P1 - P2) = K_p F$
<p>A diagram of a rectangular conductor. The left end is labeled 'T1' and the right end is labeled 'T2'. A dashed arrow labeled 'Heat flow' points from left to right above the conductor.</p>	<p>Heat flow through a conductor is proportional to the temperature difference across the ends.</p> $(T1 - T2) = K_H W$
<p>A diagram of a spring. The left end is attached to a vertical wall. The right end is attached to a horizontal line that ends in a solid arrow pointing to the right, representing the applied force 'f'.</p>	<p>Extension of a spring is proportional to the applied force.</p> $f = ke$

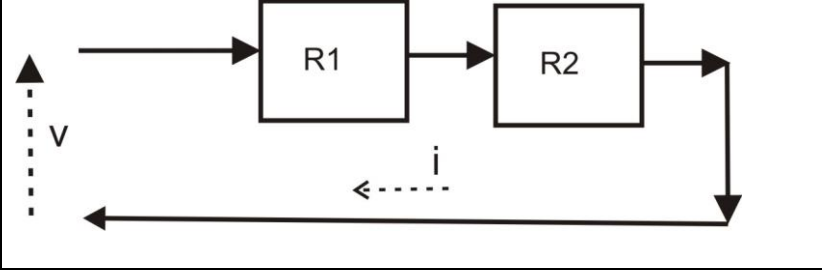

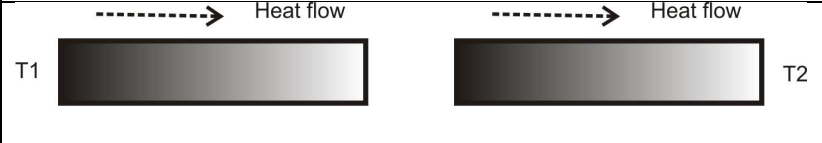
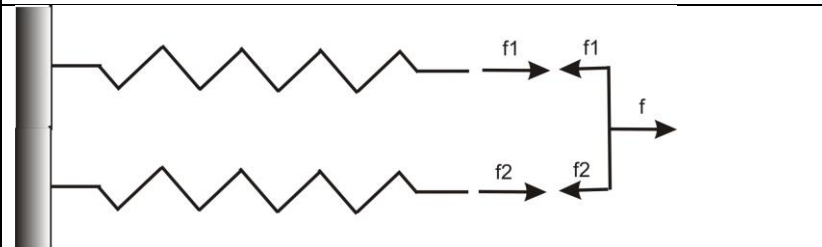
OBSERVATIONS 1

When arranged in multiples, analogous models continue to be apparent, but the analogies may be between parallel and series arrangements. If voltage (or Pressure/Temperature difference) is the distributed across many components (series arrangement) then:

$$\delta v = i \sum_j R_j; \quad \delta P = F \sum_j K_j; \quad \delta T = W \sum_j K_j;$$

A similar equation results if a force is distributed across parallel components such as springs.

$$f = x \sum_j k_j;$$

	<p>Resistors in series, the overall resistance ADDS!</p> $v1 - v2 = (R1 + R2 + \dots)i$
	<p>Pipes in series, the overall resistance to flow ADDS!</p> $P1 - P2 = (K_{P1} + K_{P2} + \dots)F$
	<p>Conductors in series, the overall resistance to heat flow ADDS!</p> $T1 - T2 = (K_{H1} + K_{H2} + \dots)W$
	<p>Springs in parallel, the overall stiffness (resistance to movement) ADDS!</p> $f = (k1 + k2)e$

REMARK: Although not discussed here, readers may like to investigate other components and scenarios which have analogous equations. Examples could include capacitors, inductors, dampers, tanks,

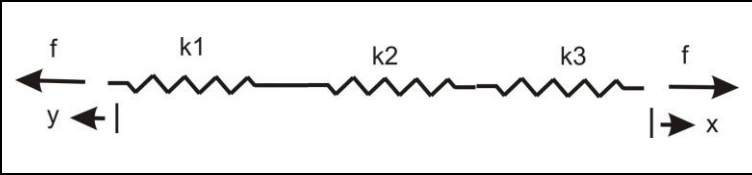
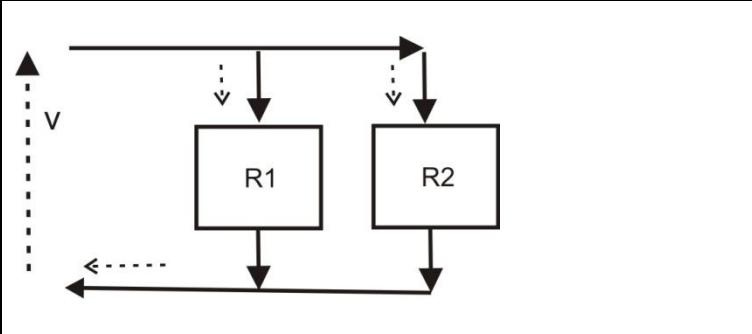
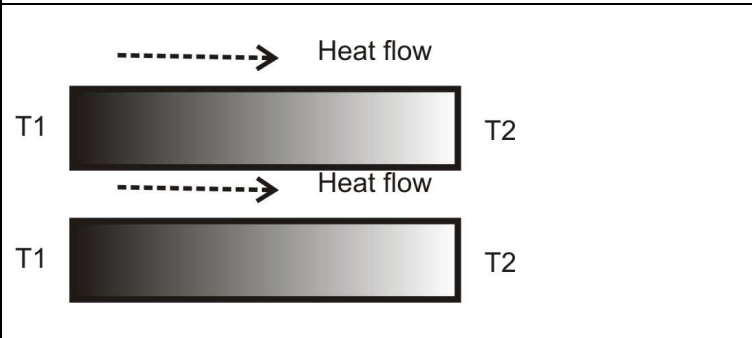
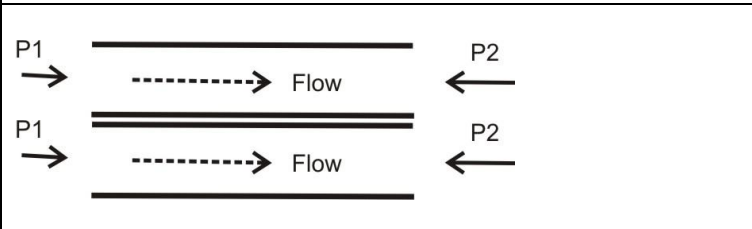
OBSERVATIONS 2:

Resistors (pipes and conductors) in parallel share the same voltage (pressure/temperature) and the current (flow) is divided among them. **Total flow is sum of flows across each component.**

$$\left\{ i = \sum_j i_j; \quad i_j = \frac{\delta v}{R_j}; \right\} \quad \left\{ F = \sum_j F_j; \quad F_j = \frac{\delta P}{K_j}; \right\} \quad \left\{ W = \sum_j W_j; \quad W_j = \frac{\delta T}{K_j} \right\}$$

Springs in series share the same force and the displacement is divided among them. Total extension is sum of extensions across each component.

$$\left\{ e = \sum_j e_j; \quad e_j = \frac{f}{k_j}; \right\}$$

	<p>Springs in series</p> $\left\{ x - y = \sum_j e_j; \quad e_j = \frac{f}{k_j}; \right\}$
	<p>Resistors in parallel.</p> $\left\{ i = \sum_j i_j; \quad i_j = \frac{\delta v}{R_j}; \right\}$
	<p>Heat conductors in parallel</p> $\left\{ W = \sum_j W_j; \quad W_j = \frac{\delta T}{K_j} \right\}$
	<p>Pipes in parallel</p> $\left\{ F = \sum_j F_j; \quad F_j = \frac{\delta P}{K_j}; \right\}$