Modelling and control summaries

by Anthony Rossiter

Modelling principles and analogies 9:

Linear systems and superposition

The first things a student must do is learn the relevant language.

Linearity when applied to systems is not the same as a straight line!

Consider a block diagram representation with system input and system output and consider the impact of different choices for input. The system can be represented by a model of the form:

$$y = f(x); \quad y = output, \quad x = input$$

The system f(x) is said to be linear if and only if the following is true:

$$f(x1+x2) = f(x1)+f(x2)$$

<u>Prove that, in general, a straight line is not a linear system</u>

Consider a typical straight line equation of the form y=mx+c and apply the test in the table above:

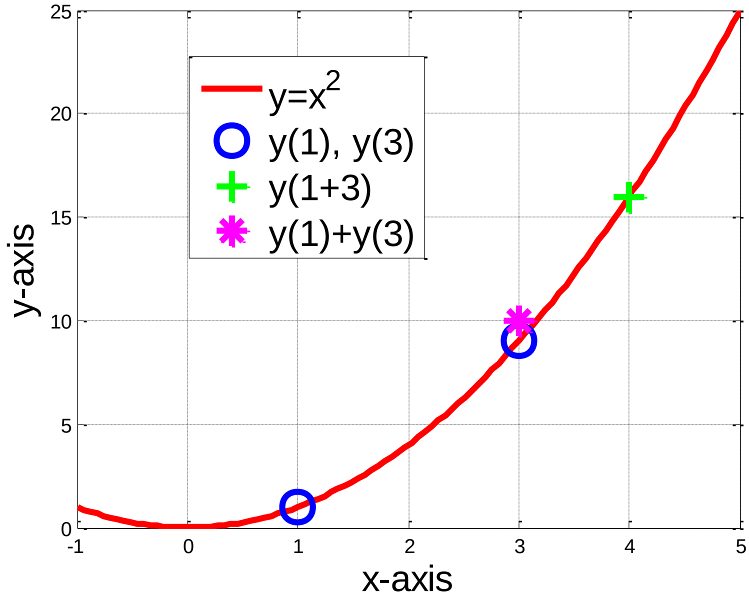
f(x1+x2)=m(x1+x2) + c whereas f(x1)+f(x2)=m x1+c+m x2 + c = m(x1+x2) + 2c

Clearly these two results are not the same UNLESS c=0!

Example of a non-linear system

It is obvious that some models (or functions) are non-linear and superposition does not apply. For example, consider a simple quadratic:

$$f(x) = x^2;$$
$$f(1) = 1;$$
$$f(3) = 9$$
$$f(4) = 16$$
$$f(1+3) \neq f(1) + f(3)$$



CONCEPTS OF SUPERPOSITION

Superposition is a very powerful concept used in linear control theory to simplify problem solving. It makes use of the core observation that f(x1+x2) = f(x1)+f(x2) and thus allows the separation of inputs (x1 + x2) into their simplest forms (x1 and x2). This process is called superposition;

We add the response to x1 to the response to x2 and this gives the response to (x1+x2). By compartmentalising responses it may be possible to simplify algebra.

APPLICATIONS WITH LAPLACE TRANSFORMS

By definition transfer functions represent linear systems and thus superposition can be used.

HENCE, given that: $$y(s) = G(s)u(s)$$

One can write:

$$y_1(s) = G(s)u_1(s); \quad y_2(s) = G(s)u_2(s)$$
$$y_{1,2}(s) = G(s)[u_1(s) + u_2(s)] = y_1(s) + y_2(s)$$

Remark

In general, the input/output relationship must be proportional for a system to be linear, that is:

output = k * input