Modelling and control summaries

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Behaviours 1 - introduction

Background - assume students are familiar with the look-up table for Laplace transforms (**see Laplace 4 for fuller table**) .

1. **A key skill is to infer directly from the Laplace transform what a signal will do and its key attributes.**
2. **Often denoted system behaviour where the signal relates to a system output.**

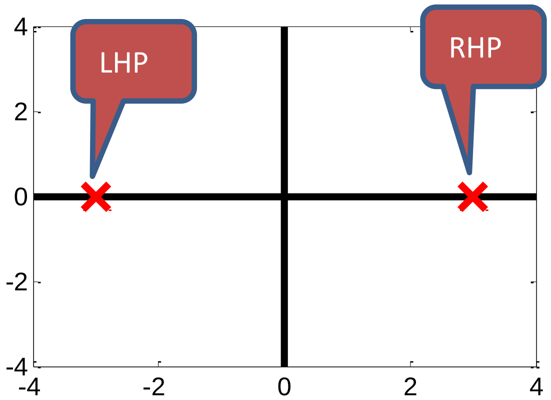
f(t)	F(s)		f(t)	F(s)
1	$\frac{1}{s}$		$\sin wt$	$\frac{w}{s^2+w^2}$
t	$\frac{1}{s^2}$		$\cos wt$	$\frac{s}{s^2+w^2}$
e^{-at}	$\frac{1}{s+a}$		$e^{-at}\sin wt$	$\frac{w}{(s+a)^2+w^2}$

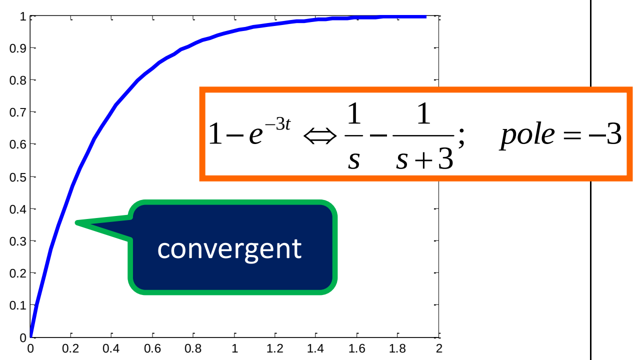
SIMPLE POLES

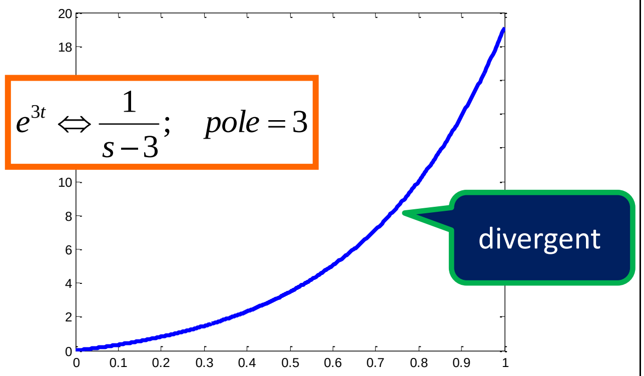
What is the link between the pole position and the time domain behaviour?

- If the pole is in the left half plane, then the exponential is convergent.
- If the pole is in the RHP, then the exponential is divergent.

$$e^{-at} \Leftrightarrow \frac{1}{s+a}; \quad e^{bt} \Leftrightarrow \frac{1}{s-b}$$







SUMMARY: Poles in LHP plane are denoted as stable and those in RHP as unstable due to the associated behaviours being convergent and divergent respectively.

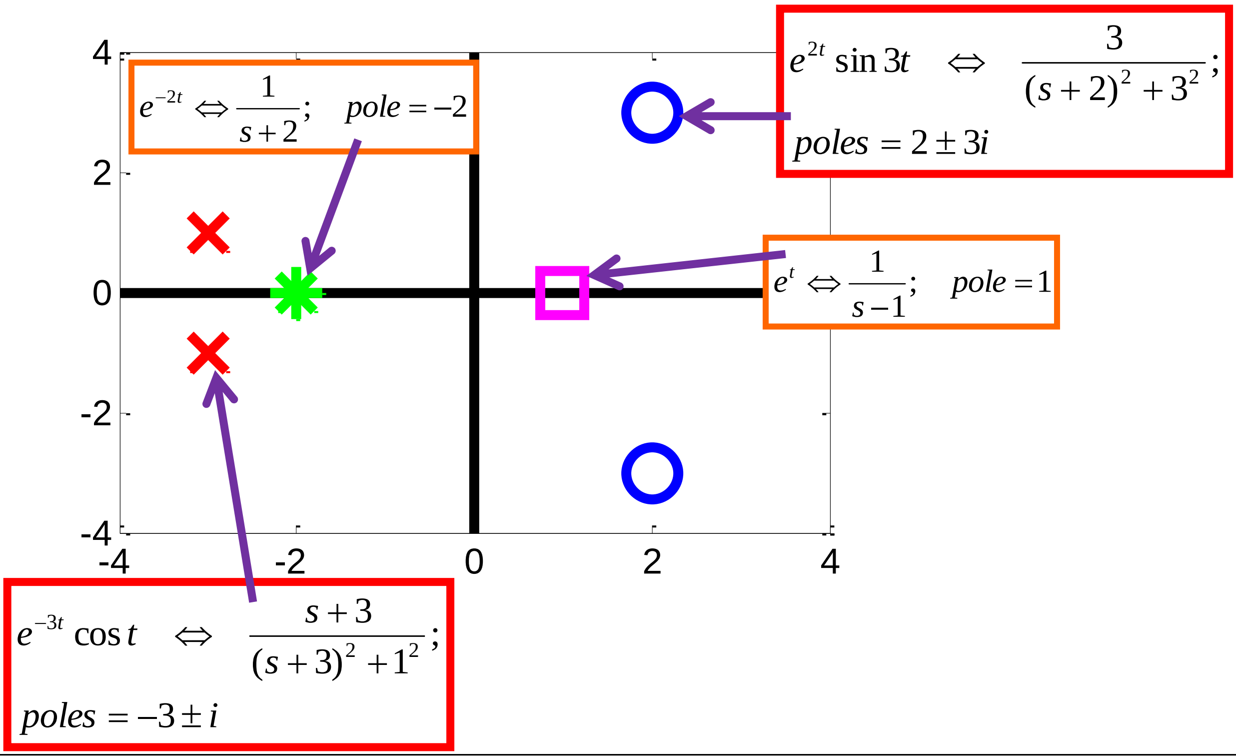
GENERALISING

- A SIGNAL/SYSTEM IS STABLE IF IT IS CONVERGENT.
- A SIGNAL/SYSTEM IS UNSTABLE IF IT IS DIVERGENT.

$e^{-bt}\cos wt \Leftrightarrow \dfrac{s+b}{(s+b)^2+w^2}$;	Convergent so stable
$te^{at} \Leftrightarrow \dfrac{1}{(s-a)^2}$	Divergent so unstable
$e^{at}\sin wt \Leftrightarrow \dfrac{s-a}{(s-a)^2+w^2}$	Divergent so unstable
$te^{-bt} \Leftrightarrow \dfrac{1}{(s+b)^2}$	Convergent so stable

Is there a link between the pole positions and whether the corresponding signal is convergent?

1. It is clear that where the poles are in the LHP, the associated signal is convergent, that is, has an exponential with a negative power.
2. Where the poles are in the RHP, the associated signal is divergent, that is, has an exponential with a positive power.
3. Note that poles on the imaginary axis (excluding origin) imply behaviour that is neither convergent or divergent in general (e.g. sinusoids oscillate).



LHP means left half plane, that is the real part is negative.
RHP means right half plane, that is the real part is positive.