Modelling and control summaries

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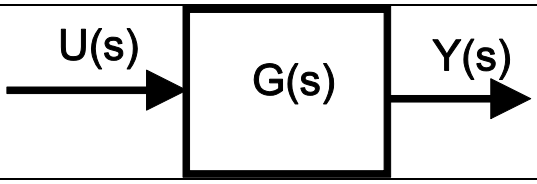
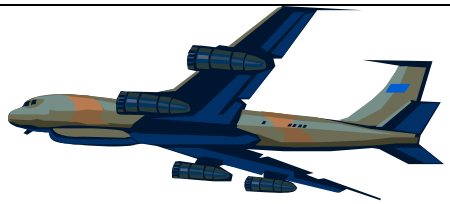
Behaviours 3 – speed of response

Assume that inferences about output behaviour Y(s) can be made be asolely on an analysis of transfer function G(s). Typically U(s) is assumed to be a step signal.	

WHAT DO I MEAN BY FAST? HOW DO I QUANTIFY SPEED OF RESPONSE

Fast change of speed would be anything less then about 20 seconds.	**A fast turn could be anything less than around a minute.**	**One would expect 0-60mph within about 4sec, so fast means 2-3 seconds!**
		

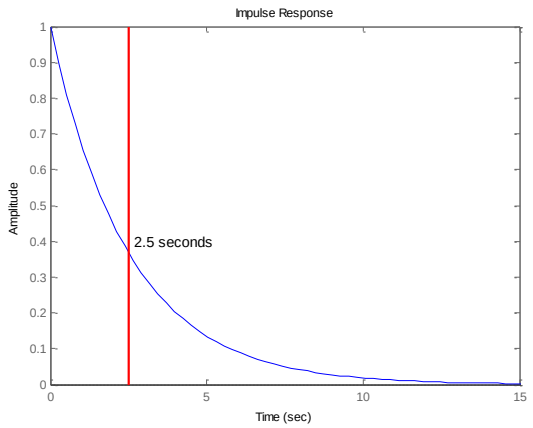
KEY OBSERVATIONS: Fast speed of response is a relative concept. First you need to define what you think is a reasonable expectation for a given context. Normal speed would match open-loop poles, for example whatever time constant comes from an underlying ODE model.

DEFINING SPEED OF RESPONSE USING FIRST ORDER MODELS

Consider the natural dynamics of a 1st order model such as:

$$G = \frac{1}{s+0.4} \equiv \frac{1}{s+\frac{1}{T}}; \quad T = 2.5$$

A time constant of 2.5 (equivalently a pole at -0.4) settles in about 7.5-10 sec (3-4 time constants).

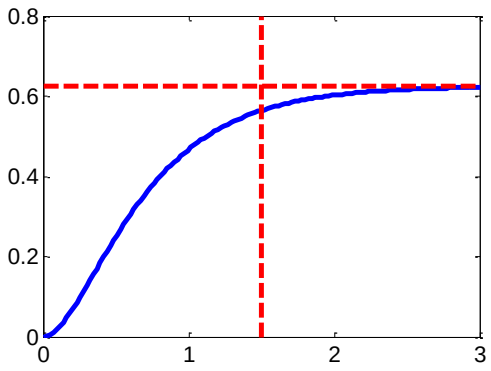
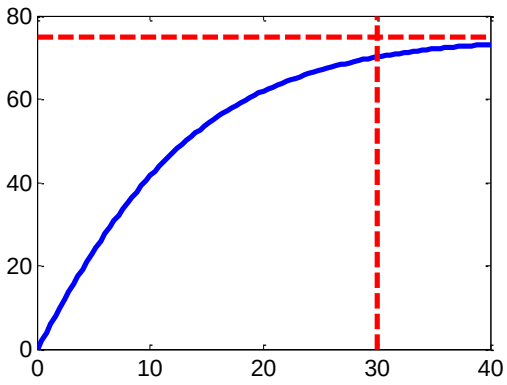
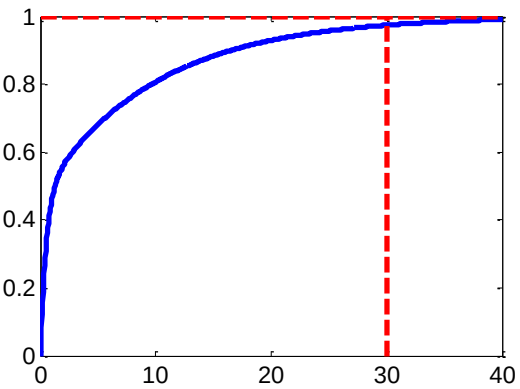


SUMMARY: If the specified settling-time is around W seconds, then we would expect that the time constant is about T=W/3 and the system pole is around pole=-3/W.
Note it is fine to approximate as this is a rather crude measure anyway.

Desired settling time	Required time constant (approx..)	Required pole (sec or hr)
120 sec or 2min	40 sec or 2/3 min	-0.025 or -1.5
0.01 sec	0.003sec	-333
2hr or 7200 sec	2400 sec or (2/3) hr	-0.0004 or -1.5

WHAT IS THE EXPECTED SPEED OF RESPONSE?

Where there are more than one pole, a default would be to assume the settling time is based on the slowest pole. In practice, it may be faster but you cannot guarantee this.

$G = \frac{5}{(s+2)(s+4)}$	Slowest pole is -2 or time constant 0.5. Expected settling is 1.5 sec. This is a fair approximation is seen here.	
$G = \frac{5(s+0.3)}{(s+0.2)(s+0.1)}$	Slowest pole is -0.1 or time constant 10. Expected settling is 30 sec. This is a fair approximation is seen here.	
$G = \frac{s+0.2}{(s+2)(s+0.1)}$	Slowest pole is -0.1 or time constant 10. Expected settling is 30 sec. This system has a much faster initial response, but the expected settling time is still a good estimate.	

SUMMARY:

1. Speed of response can sometimes be estimated from transfer function poles (time constant is inverse of pole and settling time is about 3-4 times the time constant).
2. However, if poles differ significantly in time constants, only a full partial fraction expansion will reveal residue sizes and thus contribution to behaviour of each mode.
3. In the absence of more information, the slowest mode is assumed to dominate the settling time.