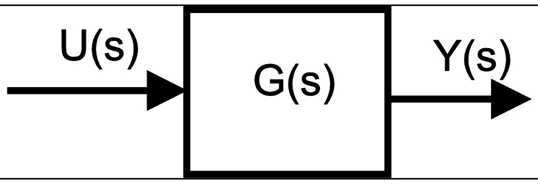
Modelling and control summaries

by Anthony Rossiter

Behaviours 4 – links to pole positions

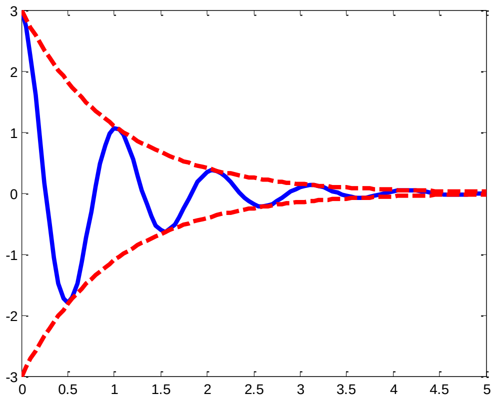
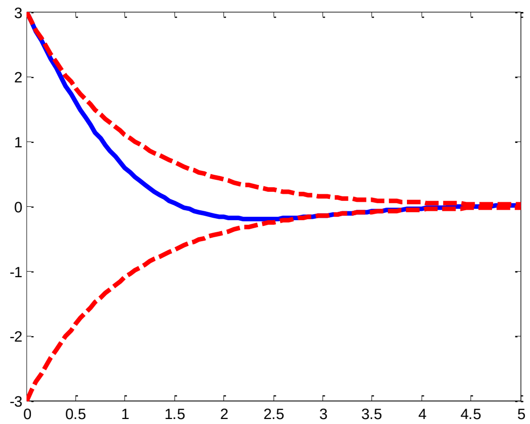
Assume that inferences about output behaviour Y(s) can be made be asolely on an analysis of transfer function G(s). Typically U(s) is assumed to be a step signal.



We know that speed of response is linked to real poles, this summary shows how the response shape in general is linked to the pole positions including those which are complex.

WHAT IS THE EXPECTED SHAPE OF THE RESPONSE?

There is an explicit link between pole positions and the corresponding time domain signal. Students should understand this link so they can infer behaviour just from a transfer function.

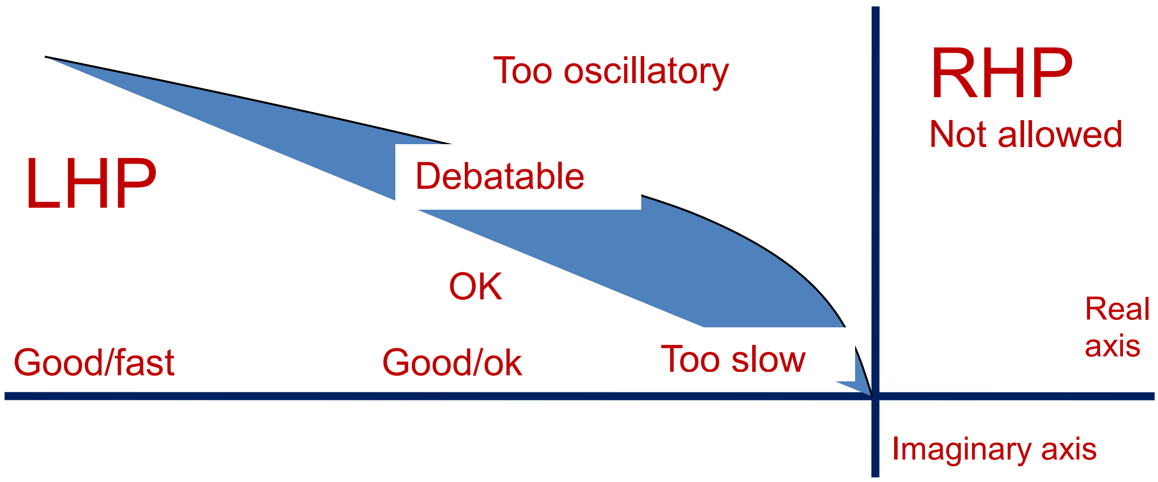
$G=\frac{3(s+1)}{(s+1)^2+6^2}$ $\Rightarrow e^{-t}(3\cos 6t)$ POLES = -1±6j Imaginary part bigger than real part.	Real part of pole is -1 or time constant 1 . Expected settling is 3 sec. Significant oscillation in transients as frequency of oscillation is 6 rad/s - taken from imag(pole).	
$G=\frac{3(s+1)}{(s+1)^2+1^2}$ $\Rightarrow e^{-t}(3\cos t)$ POLES = -1±j Imaginary part same as real part.	Real part of pole is -1 or time constant 1 . Expected settling is 3 sec. Oscillation is much slower than above (just 1 rad/s) and thus is less noticeable.	

SUMMARY: Complex poles imply oscillation – this is obvious from the table of Laplace transforms as a pole at a+jw implies a signal of the form $e^{at}\sin(wt)$. The decay is governed by the real part of the pole and the oscillation speed by the imaginary part.

WHAT CONSTITUTES ACCEPTABLE OR GOOD BEHAVIOUR?

1. Oscillation is undesirable in general, so the imaginary part should be small relative to the real part of the pole; this ensures the decay dominates the oscillation.
2. The real part should be well into the LHP to ensure fast convergence.
3. Anything on the RHP implies divergence/instability and hence is unacceptable.
4. Anything close to he imaginary axis implies the oscillation dominates decay and is unacceptable.

An indication of the likely interpretation of different pole positions is give here. The shaded blue area is one where the poles may or may not be acceptable depending upon residue sizes and context.



QUESTIONS: Using the above figure and observations, what do you **expect** from the following systems? Check your answers by doing some simulations in MATLAB noting that an expectation is not a guarantee.

$G = \dfrac{10}{s^2 + 4s + 12}$	Poles are at -2±j2√2 so imaginary part is larger. Expect the oscillation to dominate decay and therefore **expect** it to be unacceptable. However, dominance is slight and therefore this may be borderline acceptable. Expected settling time will be around 1.5 sec (time constant is about 0.5).
$G = \dfrac{3}{s^2 + 4s + 6}$	Poles are at -2±j√2 so imaginary part is larger. Expect the decay to dominate oscillation and therefore **expect** acceptable responses. Expected settling time will be around 1.5 sec (time constant is about 0.5).
$G = \dfrac{3}{(s^2 + 4s + 15)(s + 0.1)}$	Poles are at -2±j√11,-0.1 so imaginary part is larger. Expect the oscillation to dominate decay in transients and therefore unacceptable. However, expected overall settling time will be around 30 sec (time constant is about 10 for slowest mode) and oscillatory mode should decay in about 1.5 sec. so oscillations may have a smaller impact.