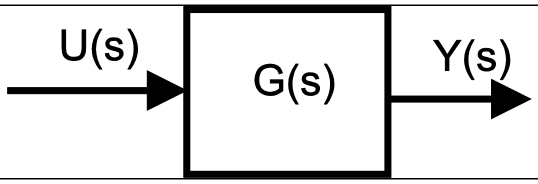
# Modelling and control summaries

by Anthony Rossiter

## Behaviours 5 – Tutorial sheet

**Assume that inferences about output behaviour Y(s) can be made be asolely on an analysis of transfer function G(s). Typically U(s) is assumed to be a step signal.**




**WHAT IS THE EXPECTED SHAPE OF THE RESPONSE?**

There is an explicit link between pole positions and the corresponding time domain signal. Students should understand this link so they can infer behaviour just from a transfer function. In this sheet students should:

1. Decide what they expect to happen.
2. Use MATLAB to test whether their expectations are met (for example using impulse.m or step.m)

**QUESTION 1** : Compare and contrast speed of response/settling time and steady-state gain for the following systems.

$$\frac{1}{s-3};\quad \frac{2}{s+1.2};\quad \frac{2000}{s+1250};\quad \frac{0.02}{s-10^{-4}};\quad \frac{0.005}{s+0.002};$$

**QUESTION 2** : Compare and contrast speed of response/settling time, transient response and steady-state gain for the following systems.

$$\frac{4.5}{s^2+s+2};\quad \frac{2}{s+1.2};\quad \frac{20}{s^2-s-2};\quad \frac{0.2}{s^2+0.25};$$

$$\frac{6}{s^2+4s+8};\quad \frac{1.5}{s^2+0.1s+1};\quad \frac{0.05}{s+0.2};\quad \frac{-3}{s^2+4s-1};$$

$$\frac{0.5}{s^2+s+4};\quad \frac{-2}{s+1.8};\quad \frac{2}{(s+1)(s^2+4s+4)};\quad \frac{12}{(s+5)(s^2+2)};$$

$$\frac{0.04}{s^2+0.2s+0.01};\quad \frac{1.5}{(s+0.1)(s^2+3s+5)};\quad \frac{0.5}{s+2};\quad \frac{3}{s^3+s^2+0.16s};$$

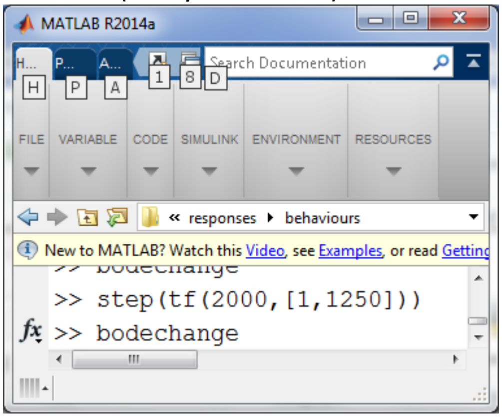
**REMARK**: Steady-state gain is discussed in analysis 1.

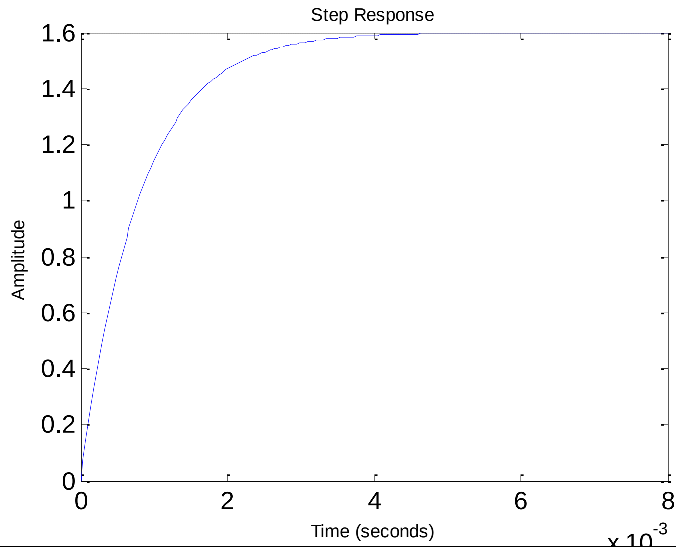
## Outline answers Q1 (taken in order):

use p for poles, T for time constant (settling time is assumed to be about 3T)

| | | | | |
|---|---|---|---|---|
| p=3 in RHP<br>unstable | p=-1.2, stable.<br>T=0.83<br>Gain=1.67 | p=-1250, stable.<br>T=0.0008<br>Gain=1.6 | p=0.0001 in RHP<br>unstable | p=-.002, stable.<br>T=500<br>Gain=2.5 |

Indicative MATLAB code to check answers (3rd system used)




## Outline answers Q2 (taken in order):

use p for poles, T for **expected** time constant (settling time is assumed to be about 3T)

| | | | |
|---|---|---|---|
| p=-0.5±j√1.75<br>stable, T=2<br>Gain=2.25<br>Expect oscillatory response | p=-1.2, stable.<br>T=0.83<br>Gain=1.67<br>Smooth | p=2,-1<br>unstable | p=±j0.5<br>Purely oscillatory<br>Gain=0.8 |
| p=-2±2j<br>stable, T=0.5<br>Gain=0.75<br>Expect damped oscillatory response | p=-0.05±j1<br>stable, T=20<br>Gain=1.5<br>Expect very oscillatory response | p=-0.2<br>stable, T=5<br>Gain=2.5<br>smooth | p=-4.23,0.24<br>unstable |
| p=-0.5±j√3.75<br>stable, T=2<br>Gain=0.125<br>Expect highly oscillatory response | p=-1.8<br>stable, T=0.56<br>Gain=-1.11<br>Smooth | p=-1,-2,-2<br>stable, T=1<br>Gain=0.5<br>Expect smooth response | p=-5, ±j√2<br>stable, T=0.2<br>Gain=1.2<br>Oscillatory mode will persist |
| p=-0.1,-0.1<br>stable, T=10<br>Gain=4<br>Expect smooth response | p=-0.1, -1.5±j√2.75<br>stable, T=10<br>Gain=3<br>Expect oscillatory response but fast decay compared to other mode. | p=-2<br>stable, T=0.5<br>Gain=0.25<br>smooth | p=0,-0.2, -0.8<br>integrator mode<br>T=5 if convergent<br>Gain=∞<br>Converges only if input tends to zero. |