<u>Easy questions on 1st order responses</u>

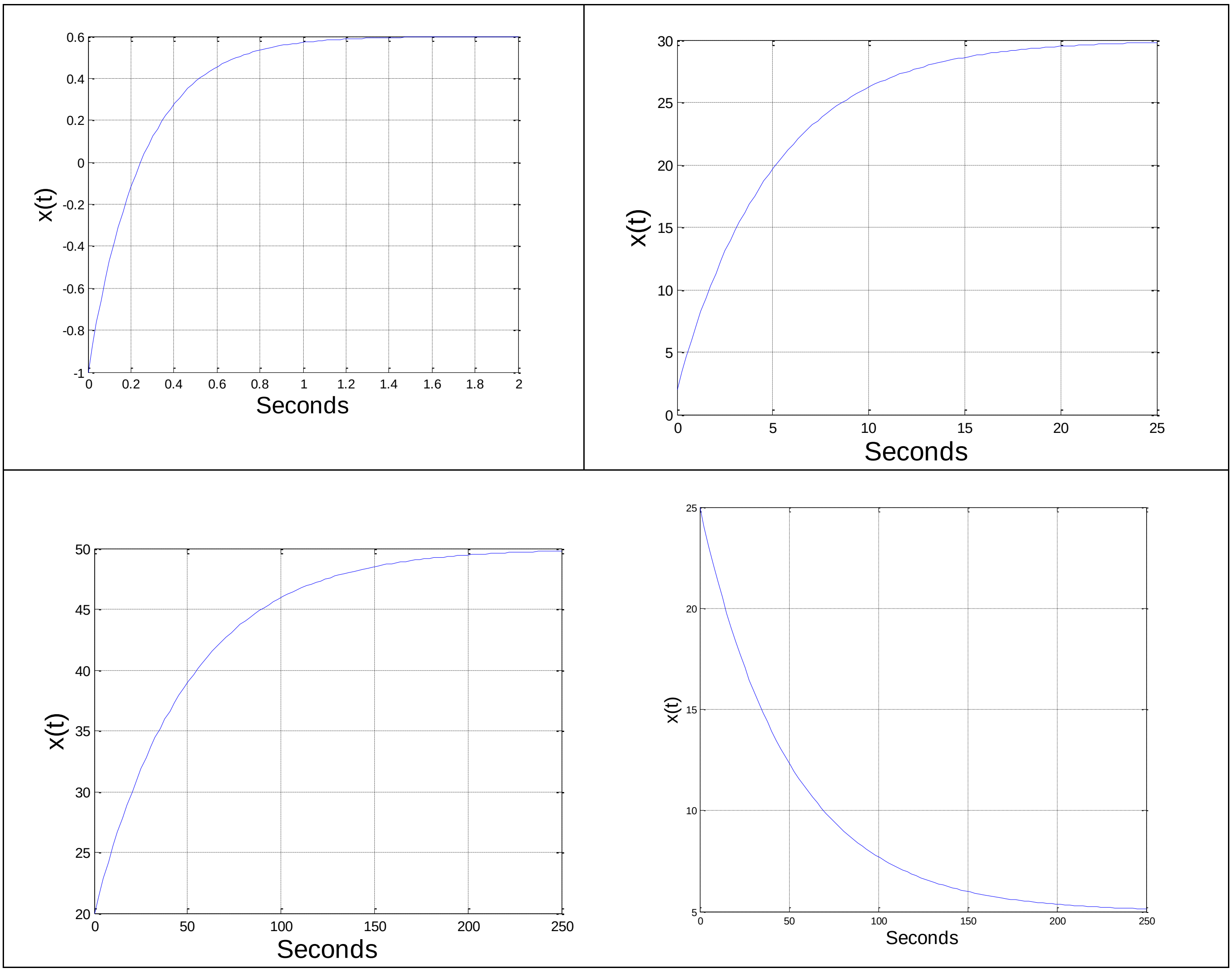
Q1. Sketch the responses for the following 1st order ODEs without solving explicitly.

$$\left\{2\frac{dx}{dt}+5x=0; \quad x(0)=-4\right\} \qquad \left\{20\frac{dy}{dt}+4y=0 \quad y(0)=1\right\}$$

$$\left\{0.01\frac{dw}{dt}+0.005w=3; \quad w(0)=200\right\} \qquad \left\{2\frac{dz}{dt}-5z=2; \quad z(0)=-1\right\}$$

Q2: Solve the ODEs of Q1 explicitly and hence generate precise sketches. Contrast these with the sketches you made for Q1.

Q3: Estimate the parameters of the underlying 1st ODE from the following sketches. You can assume that the systems are excited by a unit step input. How would your answers differ if the input had magnitude 2.5?



Q4: Determine the components of an RC circuit which will give a settling time (to within 2%) of 20ms and draw a maximum current of 4mA with a supplied voltage of 5V.

Q5: The acceleration of an aircraft on the runway can be approximated by a mass damper system. Given a mass of 200 ton and effective damping of 20000Ns/m while on the runway, determine the velocity of the aircraft and in particular, what engine force is required to accelerate to 150mph within 30 sec. [Hint: first change mph to m/s]

Q6: A medical device for needle placement can be well approximated by a spring-damper and with negligible mass. Given the desired displacement distances are of the order of 1-4cm and damping is typical 100Ns/m, design a spring and specification for the required input force to give a suitable gain and dynamics which settle in the order of 2 seconds. Sketch the response of this system.

Q7. Compute the time constant and gains.

$$2f = 4\frac{dv}{dt} + 3v \qquad 2u = 30\frac{dh}{dt} + \frac{1}{3}h$$

$$0.1f = 0.3\frac{dx}{dt} + 2x \qquad 1.53u = 1100\frac{dv}{dt} + 170v$$

Q8. Determine the responses for the following systems. In all cases assume u(t) is a unit step.

$$4\frac{dx}{dt} + x = 2u; \quad x(0) = -2$$

$$0.5\frac{dz}{dt} + 0.2z = 3u; \quad z(0) = 10$$

$$100\frac{dy}{dt} + 20y = 50u; \quad y(0) = 1$$

Q9. Find x(t) for t=0, T, 2T, 3T, 4T

$$T\frac{dx}{dt} + x = 0 \quad \Rightarrow \quad x(t) = x(0)e^{-\frac{t}{T}}$$

$$0.2\frac{dx}{dt} + 0.4x = 1 \quad \Rightarrow \quad x(t) = 2.5(1 - e^{-2t})$$

Q10. The temperature *T* in a house is given by:

$$A\frac{dT}{dt} + k(T - T_e) = h$$

a) Given the settling time to 5% of steady-state is 2 hours and the steady-state internal temperature, with maximum heating *h=1* and an external temperature T_e of -5 degree, is 25 degrees, hence determine the model parameters *A,k*.

b) Sketch the expected temperature response for an initial temperature of 5 degrees, an external temperature of 0 degrees and 60% of full heating.

c) Take Laplace transforms of the system and hence show how Laplace techniques can give the same solution in time as determined in part (c).

Q11. Sketch the expected responses for the following systems. Assume u(t) is a unit step. Use MATLAB to test your answers.

$$0.5\frac{dz}{dt}+0.2z=3u; \quad z(0)=10$$

$$2\frac{dx}{dt}+6x=4u; \quad x(0)=-1$$

$$4\frac{dx}{dt}+x=2u; \quad x(0)=-2$$

Q12: The model of a climbing aircraft was derived as follows. With initial condition z(0)=0, sketch z(t).

$$\frac{200}{300}\frac{dz}{dt}+z=\frac{17.75}{3}$$

Q13: The speed of a car is well represented as a mass-damper system. Given a Mass of 800kg, a damping coefficient of 200Ns/m and an applied force of 4800N, sketch the velocity response for an initial velocity of 10m/s.

Q14: A large transport company wishes to design a control system for the temperature inside an aeroplane while at cruising altitude. The target internal temperature is 20 degrees while the external temperature is typically about -50 degrees. Heat loss through the airframe is estimated is about 1kW/degree and the heat input from the passengers is about 200W per passenger. The overall heat capacity is approximated by 1MJ/deg. The heating available is uniformly distributed and is given as 2kW/volt where the input signal is supplied in volts.

a) Derive a model for how the temperature in the cabin is linked to the voltage input with 100 passengers and hence show that a voltage of about 34 volts is good in general.

b) Plot a temperature profile if the external temperature drops suddenly by 10 degrees but the voltage is unchanged.

c) Discuss the temperature profile if people in the aeroplane suddenly generate 20% more heat than expected – you can begin from an assumed steady-state of 20 degrees.

OUTLINE ANSWERS

Q1,2: Bookwork straight from notes: Use MATLAB to generate solutions and check against you work.

e.g. for 1st of these

```
x=dsolve('2*Dx+5*x=0','x(0)=-4');
t=linspace(0,2,100);
xt=subs(x,t);
figure(1);clf reset
plot(t,xt,'b');grid
```

Q3: Throughout Q3 assume a model of the form $T\frac{dw}{dt}+w=ku(t)$.

Steady-state is 0.6. Initial value is -1. Rise is given as 1.6.
63% of rise is given by 0.63*1.6=1.008 which implies x(t)=0.008. x(t) has this value at about t=0.25 sec and therefore T=0.25, k=0.6.

Steady-state is 30. Initial value is 2. Rise is given as 28.
63% of rise is given by 0.63*28=17.64 which implies x(t)=19.64. x(t) has this value at about t=5 sec and therefore T=5, k=30.

Steady-state is 50. Initial value is 20. Rise is given as 30.
63% of rise is given by 0.63*30=18.9 which implies x(t)=38.9. x(t) has this value at about t=50 sec and therefore T=50, k=50.

Steady-state is 5. Initial value is 25. Drop is given as 20.
63% of drop is given by 0.63*20=12.6 which implies x(t)=12.4. x(t) has this value at about t=50 sec and therefore T=50, k=5.

Q4: Maximum current is at t=0 and given as V/R: Therefore R=V/I = 5/0.004 = 1250ohms.
Time constant is given by RC, so T=5ms implies that C=.005/1250 = 4 microF.

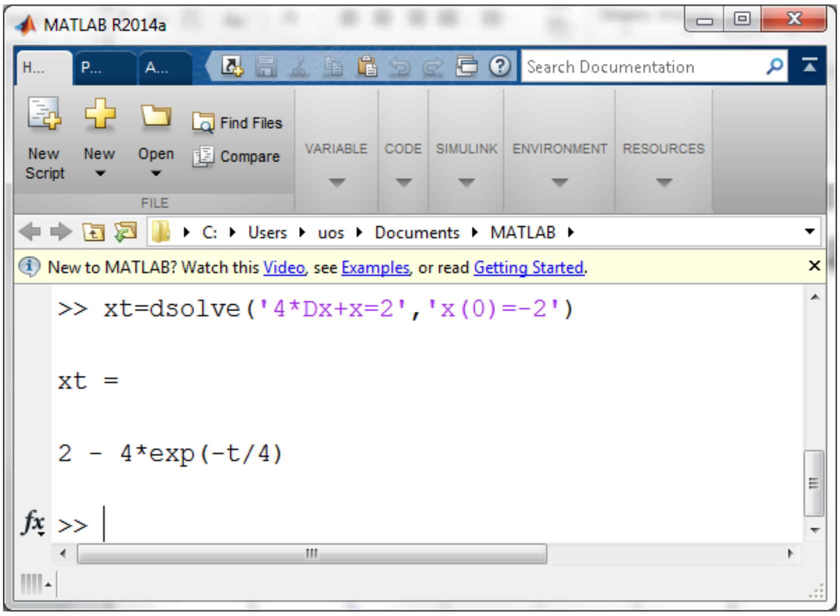
Q5: Parameters give a time constant of 10sec so after 30 sec aircraft at 95% of steady-state land speed.
1mph is the same as 1609metre ph or (1609/3600)m/s
150mph is the same as 67m/s

Steady-state is given as f/B. Therefore min f required is 67B [scaled by (1/.95) to be precise].

Q6: Model is Bdx/dt +kx =f   or   (B/k) dx/dt +x =f/k
Desired time constant is about 0.8sec, therefore (B/k)=0.8 so k=125N/m
Steady-state displacement is given as (1/k)f = 0.04, and therefore f=6N is required.

Q7: Put in to time constant form that is   T(dx/dt) + x = ku and then the answers should be obvious:
i) T=4/3, k=2/3 ;   ii) T=90, k=6; iii) T=0.15, k=0.05;   iv) T=110/17, k=1.53/170

Q8: Check your answers with MATLAB, for example the first one can be solved using



Q9: Straight from notes

Q10: A 5% offset is equivalent to 3 time constants, so the time constant is 2/3 hours. Next, substitute values given so:

$$\left\{\frac{A}{k}\frac{dT}{dt} + T = \frac{1}{k}h + T_e,\ \frac{dT}{dt} = 0\right\} \quad \Rightarrow \quad \left\{25 = \frac{1}{k} - 5\right\}$$

Hence k=1/30 and (A/k)=2/3 so A=2/90.
Thereafter this is bookwork. 60% heating implies that h=0.6 so the steady-state difference between internal and external temperatures will be 0.6*30=18.

Q11,12: Bookwork. Similar to earlier questions so use MATLAB to check working.

Q13: Need to show that model is given as follows, and bookwork thereafter:

$$M\frac{dv}{dt} + Bv = f; \quad M = 800, \quad B = 200, \quad f = 4800, \quad v(0) = 10$$

Q14: First derive a model for the temperature using heat capacity equations

$$10^6 \frac{d\vartheta}{dt} = 1000(\vartheta_e - \vartheta) + 200 \times 100 + 2000v; \quad \vartheta_e = -50$$

In order to get a steady-state of 20, then:

$$0 = 1000(-50 - 20) + 20000 + 2000v \quad \Rightarrow \quad v = \frac{50000}{2000} = 25$$

b) The effective time constant model is given as:

$$10^6 \frac{d\vartheta}{dt} + 1000\vartheta = \underbrace{1000\vartheta_e + 20000 + 2000v;}_{heat\,nput}$$

A drop of 10 degree in external temperature gives a step change in heat input so sketch is simple bookwork.

c) Again, this just gives a change in the expected overall heat input. Now the term 20000 is increased by 20% to 240000 so the steady-state will rise by 4.