More advanced questions on 1st order responses

1. A car manufacturer wants to design a simple cruise control for their new model. Early investigations are based on a simple model for the vehicle dynamics given as:

$$M\frac{dv}{dt}+Bv=u$$

where v is velocity and u is linked to the throttle position, M=1000 and B=100. The cruise control introduces a simple feedback term given as u=K(r-v), where r is the desired speed. Hence

$$M\frac{dv}{dt}+Bv=K(r-v) \quad \Rightarrow \quad M\frac{dv}{dt}+(B+K)v=Kr$$

Investigate the behaviour of this system for various values of K (e.g. 100, 500) and in particular comment on whether one can ever achieve the desired speed (take r to be a step) with this scheme. Also discuss the impact of differing K on throttle usage.

2. A tank level control system is designed such that the tank will hold 1000 litres of water at full depth with a corresponding pressure at the bottom of 0.2 atmospheres (about $20000N/m^2$). Use the data supplied to determine the cross-sectional area and full depth. [You can take gravity to be $10ms^{-2}$]

- With no inflow and an open output plug, the tank should have discharged 95% of its contents in 150 seconds. Assuming that with the plug open, the dynamics are adequately represented by a 1st order model, determine an appropriate model for this scenario.
- With a closed-plug and a full inflow, the tank should fill in 100 seconds. By finding the implied rate of change of depth with the plug closed, hence determine a model to represent the system with a full inflow and an open plug. [HINT: Show that the steady-state depth will be at h=1m].

Sketch the depth response with the plug open and a full inflow starting from an initial depth of 0.6m.

3. A pulley has inertia $J=4kgm^2$ and is subject to a torque 3Nm. What is the damping B if it reaches a rotational speed of 2 rad/s after 3.2sec. (starting from zero angular velocity) ? The model is given as

$$J\frac{dw}{dt}+Bw=\tau$$

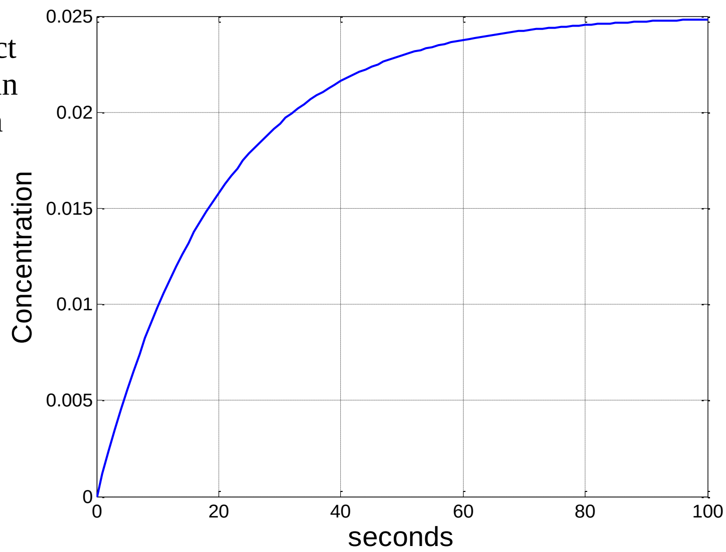
4. A resistor capacitor circuit with, no voltage supply, is required to discharge 0.4Coulombs in 0.01sec with a current that varies no more than 20% from its initial voltage storage of 3V. Give some possible circuit parameters? The model is given as:

$$R\frac{dq}{dt}+\frac{q}{C}=v; \quad \frac{dq}{dt}=i$$

5. The concentration *C(t)* ($mole/m^3$) of the product of a continuous chemical reaction is affected by an input *F(t)* (a flow rate in m^3/s). The concentration can be modelled by a 1st order differential equation with unknown constants b, d.

The technician performs an experiment by giving a step change in *F(t)* equal to $0.005m^3/s$.

The concentration then follows the pattern given in figure here. Use this data to estimate the model parameters for this process.

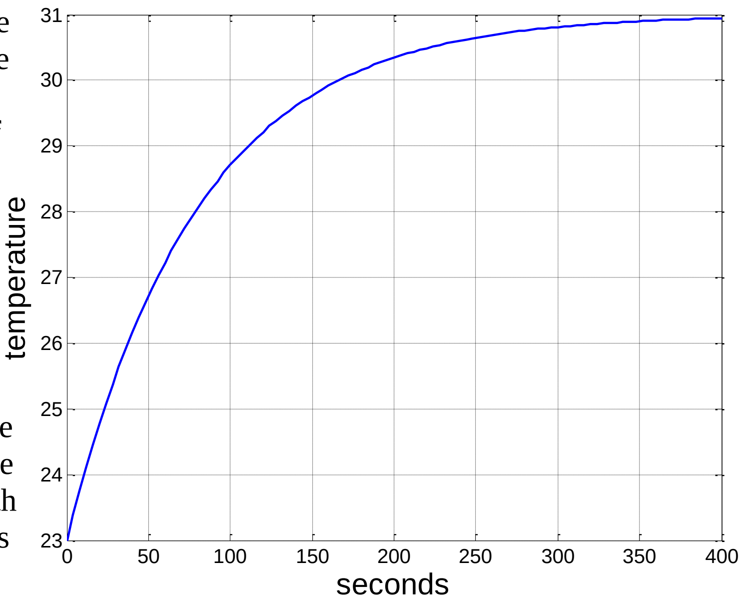


Sketch the expected behaviour if the flow is changed from a steady $0.005m^3/s$ to $0.008m^3/s$.
What would be the effect on the behaviour of adding a catalyst which increases the rate of production of C by $0.01mole/m^3s$?

6. Sketch the speed response expected of a 1 ton car with an engine force of 8000 Newton onto the road. If the steady-state speed is 30m/s, what is a good estimate of the frictional force? How would the speed curve change if the initial speed was 10m/s or 40m/s?

7. A block of mass 3kg (specific heat capacity 500J/kg/deg) is heated with a 1KW heater. Sketch the temperature response expected. If the steady-state temperature is 150 degrees, what is a good estimate of the heat loss coefficient? How would the temperature curve change if the initial temperature was 50 degree or 200 degree?

8. A heating system is initially at a steady-state of 23^oC with a heating input of 2kW. The response in temperature (^oC) to a step change (increase) in the input of magnitude 3.2kW is given the figure here.

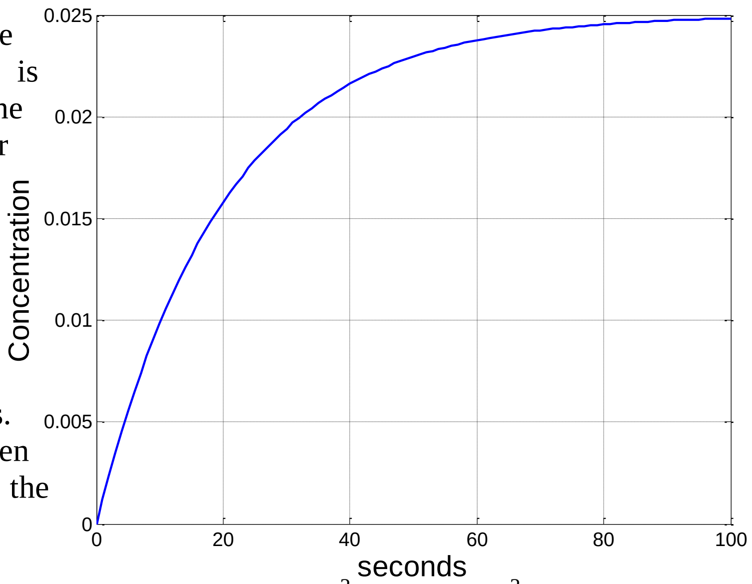


Use this data to estimate a model for this process.

Estimate the temperature of the surroundings and hence determine and sketch the expected behaviour if the surrounding temperature suddenly drops to minus 10^oC. Assume one begins from a steady-state temperature with heating input of 5.2kW and this heating is retained.

9. The concentration *C(t)* ($mole/m^3$) of the product of a continuous chemical reaction is affected by an input *F(t)* (a flow rate in m^3/s). The concentration can be modelled by a 1st order differential equation with unknown constants b, d.

$$\frac{dC}{dt} + bC = dF$$



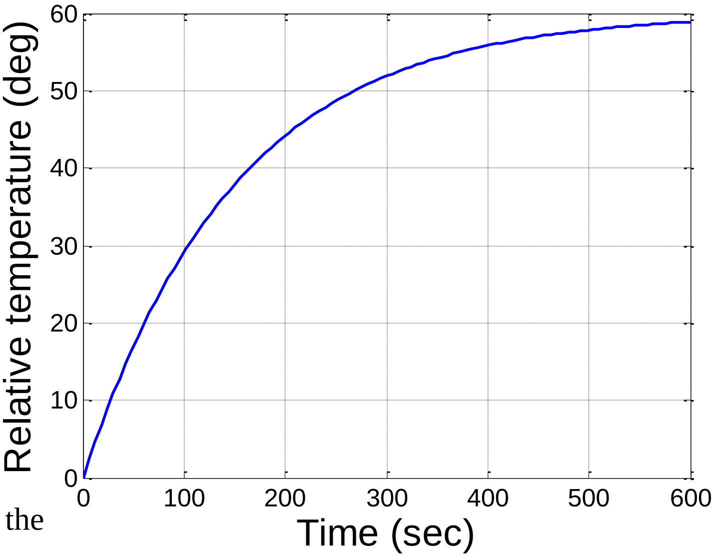
The technician performs an experiment by giving a step change in *F(t)* equal to $0.005m^3/s$. The concentration then follows the pattern given in the figure here. Use this data to estimate the model parameters for this process.

Sketch the expected behaviour if the flow is changed from a steady $0.005m^3/s$ to $0.008m^3/s$.

What would be the effect on the behaviour of adding a catalyst which increases the rate of production of C by $0.01mole/m^3s$?

10. The temperature T*(t)* (relative to external temperature) of the product in a manufacturing process is affected by an input heating h*(t)* (watts); heating is provided by burning fuel with a heat output of 100kW/kg. Fuel is supplied at a rate R kg/s. The temperature can be modelled by a 1st order differential equation with unknown constants b, d.

$$\frac{dT}{dt} + bT = dh$$



The technician performs an experiment from zero initial conditions by giving a step change in *h(t)* equal to 12kW. The temperature then follows the pattern given in the figure.
Use this data to estimate the model parameters for this process.

Hence find the parameters of a model in time constant form where the input is fuel flow *R* rather than heating power *h*.
Sketch the expected behaviour if the fuel flow *R(t)* is changed from a steady 0.1kg/s to 0.04kg/s.
Discuss the likely impact of a change in the calorific value of the fuel?

ANSWERS:

The intention is that you work hard at applying your core knowledge until you can do these unaided and also are able to state confidently that you have the answer correct. For feedback on your attempts, come to a tutorial session with your detailed working.

The usual technique is to begin from what you know, apply that and hence find out what you can, even if this does not seem to answer the question directly. This means the first step is to write down, in brief summary, what you know about 1st order system responses and write this alongside the information provided.

Once you have added to the information provided by doing simple computations, it will be more obvious how to answer the specific questions.