Modelling and control summaries

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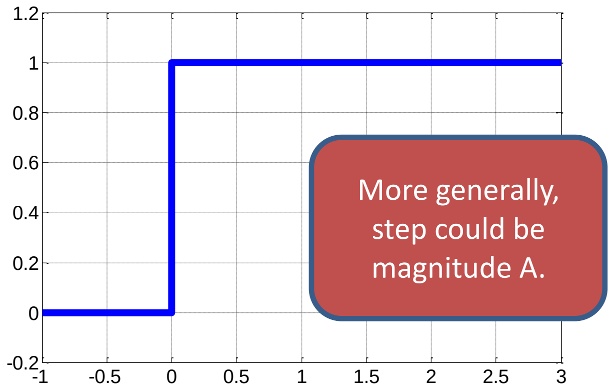
1^{st} order responses 2: Step response

Assume a standard time constant form for a 1^{st} order model with constant coefficients, output y(t) and input u(t).	$T\frac{dy(t)}{dt}+y(t)=K\,u(t)$

STEP RESPONSE: A step in the input u(t) corresponds to sudden changes in the input value: (i) a tap being switched on; (ii) a switch going from off to on; (iii) a heater having the kW setting changed, etc. For convenience, compute the unit step response with zero initial conditions.

$$\left\{\begin{matrix} u(t)=0 & t<0 \\ u(t)=1 & t\ge 0\end{matrix}\right\} \qquad L[u(t)]=\frac{1}{s}$$



OUTPUT STEP RESPONSE (u(t)=1 and x(0)=0)

To confirm this solution, substitute *x(t)* into the original differential equation and verify that it satisfies both the equation and the initial condition that *x(t)=0* when *t=0.*

$$T\frac{dx}{dt}+x=K\times 1 \quad\Rightarrow\quad x(t)=K(1-e^{-\frac{t}{T}})$$

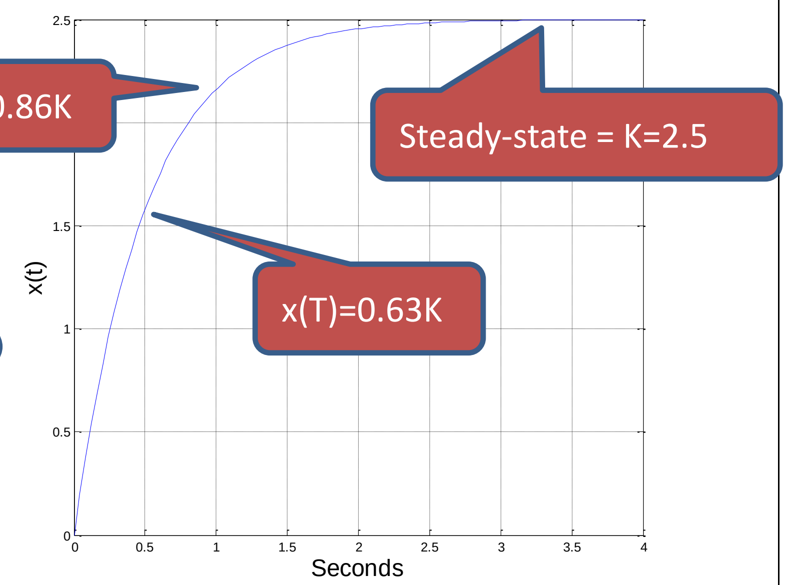
Verify solution using Laplace transforms

$$\left.\begin{matrix} L[Ku]=K/s \\ L[x(t)]=X(s) \\ L[\frac{dx}{dt}]=sX(s)\end{matrix}\right\} \Rightarrow \begin{matrix}[sT+1]X(s)=K/s \\ \Downarrow \\ X(s)=\frac{K/T}{s(s+1/T)}=\frac{K}{s}-\frac{K}{s+1/T}\end{matrix}$$

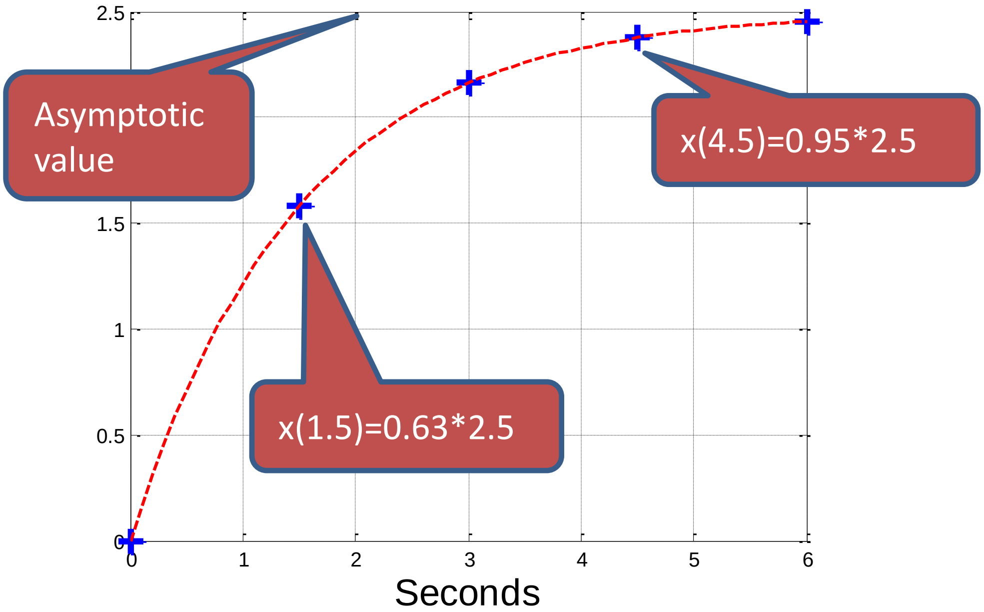
$$x(t)=K(1-e^{-\frac{t}{T}})$$
$$x(T)=K(1-e^{-1})$$
$$x(3T)=K(1-e^{-2})$$
$$x(3T)=K(1-e^{-3})$$
$$1-e^{-1}=0.63,1-e^{-2}=0.86$$
$$1-e^{-3}=0.95,1-e^{-4}=0.98$$

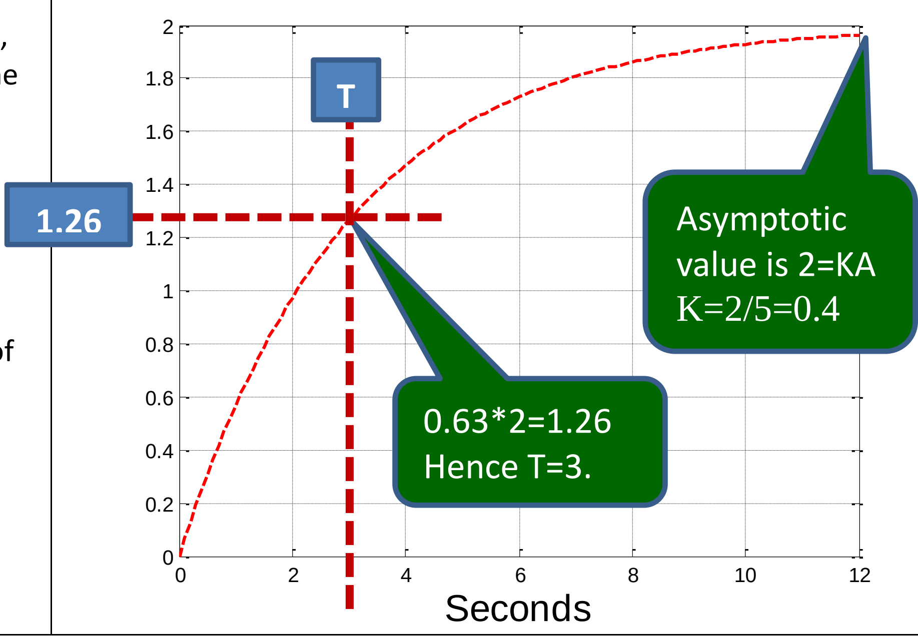
Do example

$$0.5\frac{dx}{dt}+x=2.5u; \quad T=0.5$$



The rate of convergence to steady-state is explicitly linked to the time constant T.

Sketch the step response for:	$\left\{ 6\frac{dx}{dt} + 4x = 5u; \quad u = 2 \right\} \quad \Rightarrow \quad 1.5\frac{dx}{dt} + x = 1.25u$
1. Find the time constant. Here T=1.5. 2. Find system gain K (here 1.25) and note input magnitude A (here 2). Hence KA=2.5 3. Calculate x(T), x(2T), x(3T) and draw a smooth curve between them. x(T)=0.63*K*A x(2T)=0.86*KA x(3T)=0.95*KA	

PARAMETER ESTIMATION	Given that the input magnitude A= 5, find the gain K and time constant T.
Given the plot has a known form for a first order model, one can easily determine the model parameters from a step response curve. For system gain K, input magnitude A then: • Steady-state =KA • Time to move 63% of steady-state is the time constant T. **HENCE, model is** $3\frac{dx}{dt} + x = 0.4u$	

First order step responses are characterised by simple observations.

1. Response begins at zero and finishes at gain*input magnitude.
2. Use multiples of T on the time axis, then the curve is invariant (0% at 0 sec, 63% at T sec, 86% at 2T sec, 95% at 3T sec, etc.).