Modelling and control summaries

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1^{st} order responses 3: Complete

Assume a standard time constant form for a 1^{st} order model with constant coefficients, output y(t) and input u(t).	$T\dfrac{dy(t)}{dt}+y(t)=K\,u(t)$

SUPERPOSITION: As the system is linear, one can determine the response for a step input with a non-zero initial condition simply by adding the two separate responses together.

$$T\frac{dx}{dt}+x=Ku \quad\Rightarrow\quad x(t)=Ku(1-e^{-\frac{t}{T}})+x(0)e^{-\frac{t}{T}}=Ku+(x(0)-Ku)e^{-\frac{t}{T}}$$

Step response (no initial condition)

Response with no input

Verify this result for yourself by taking Laplace and deriving from first principles.

$$L[T\frac{dx}{dt}+x]=L[Ku] \quad\Rightarrow\quad sTX(s)+X(s)-Tx(0)=\frac{Ku}{s},$$ **etc.**

(u taken to be magnitude of step input)

1^{st} order step response summary

1. Clear link between behaviour and parameters.
2. Clear link between model form and behaviour.
3. Modelling

1. Could choose parameters to achieve desired behaviour.
2. Systems giving this model form have known behaviour.
3. Can determine model parameters from the behaviour

$$T\frac{dx}{dt}+x=Ku;\quad u(s)=\frac{A}{s}$$

$$x(t)=(x(0)-KA)e^{-\frac{t}{T}}+KA$$

Steady-state =KA

Distance of movement is difference between initial condition and steady-state.

Convergence rate to steady-state depend- solelv on T

QUESTION 1 Determine the response for	$\left\{6\frac{dx}{dt} + 4x = 3u; \quad u = 1; \quad x(0) = 2\right\}$
1. Put into time constant form	$\left\{1.5\frac{dx}{dt} + x = 0.75u; \quad u = 1; \quad x(0) = 2\right\}$
2. Determine K, A and T	By inspection: K=0.75, A=1, T=1.5 and KA=0.75
3. Substitute into known solution	$\left\{x(t) = (x(0) - KA)e^{-\frac{t}{T}} + KA\right\} \Rightarrow \left\{x(t) = (2 - 0.75)e^{-\frac{t}{1.5}} + 0.75\right\}$

QUESTION 2 Determine the response for	$\left\{0.2\frac{dx}{dt} + \frac{x}{3} = 0.45u; \quad u = 2; \quad x(0) = -2\right\}$
1. Put into time constant form	$\left\{0.6\frac{dx}{dt} + x = 1.35u; \quad u = 2; \quad x(0) = -2\right\}$
2. Determine K, A and T	By inspection: K=1.35, A=2, T=0.6 and KA=2.7
3. Substitute into known solution	$\left\{x(t) = (x(0) - KA)e^{-\frac{t}{T}} + KA\right\} \Rightarrow \left\{x(t) = (-2 - 2.7)e^{-\frac{t}{0.6}} + 2.7\right\}$