Modelling and control summaries

by Anthony Rossiter

1st order responses 4: Sketching

Assume a standard time constant form for a 1st order model with constant coefficients, output y(t) and constant input u(t).

$$\left\{\begin{matrix} T\dfrac{dx}{dt}+x=Ku(t) \\ u(t)=A, t\geq 0\end{matrix}\right\} \Rightarrow \quad x(t)=KA(1-e^{-\frac{t}{T}})+x(0)e^{-\frac{t}{T}}=KA+(x(0)-KA)e^{-\frac{t}{T}}$$

steady-state

Distance from steady-state

COMPUTE the values at integer multiples of the time constant using fractions of the initial distance from steady-state d= x(0)-KA

$$x(T)=KA+d\times 0.37$$
$$x(2T)=KA+d\times 0.14$$
$$x(3T)=KA+d\times 0.05$$

Moved 63% in one time constant (or 37% still to go).

Only 5% from steady-state after 3 time constants.

BASIC TECHNIQUE: Mark the values x(0), x(T), x(3T) and the steady-state and draw a smooth line between these.

EXAMPLE 1 *(Assuming that the input is a unit step)*

$$2\frac{dx}{dt}+6x=4u \quad \Rightarrow \quad T=\frac{1}{3}, \quad KA=\frac{4}{6}$$
$$u(t)=1,\ x(0)=-1$$

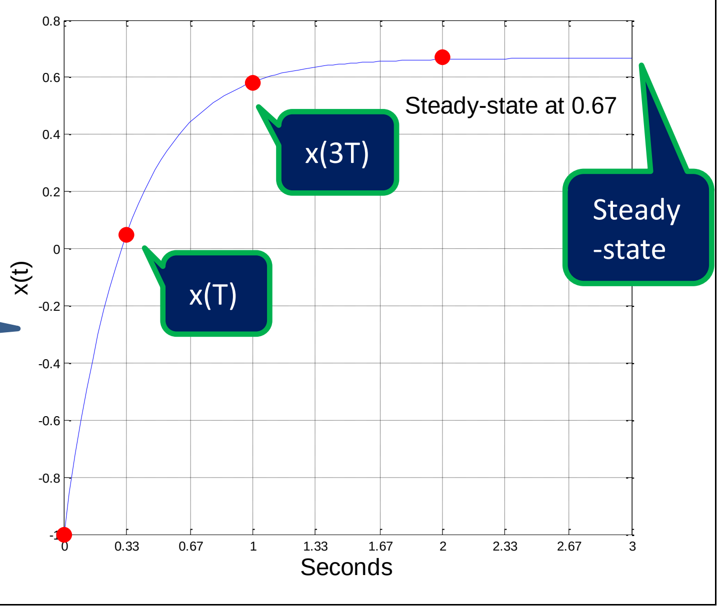
PRELIMINARY STEPS

1. Steady-state is 2/3
2. Initial offset d=x(0)-KA = -5/3

Compute x(T) and x(3T).

$$x(T)=KA+d\times 0.37=\frac{2}{3}-0.37\times\frac{5}{3}=0.05$$
$$x(3T)=KA+d\times 0.05==\frac{2}{3}-0.05\times\frac{5}{3}=0.58$$

Place data on curve and join by a smooth curve.



IMPROVING SKETCH WITH A GRADIENT

The initial gradient can be computed by inspection and thus added to ensure the sketch is more accurate. **NO computation is needed!**

At t=0, find the gradient using simple differentiation.

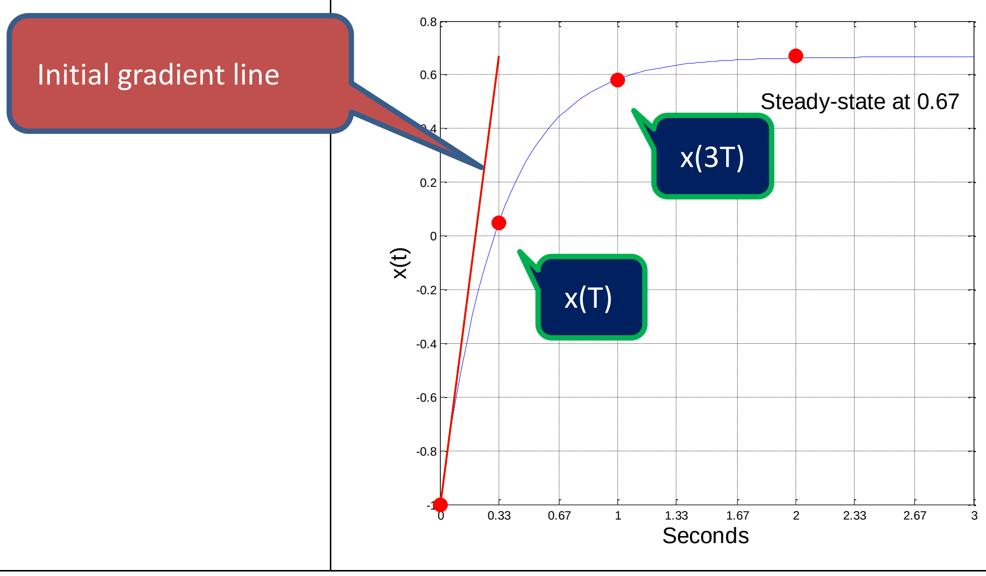
$$\left.\begin{aligned} x &= (x(0)-KA)e^{-\frac{t}{T}} + KA \\ \frac{dx}{dt} &= \frac{(x(0)-KA)}{-T}e^{-\frac{t}{T}} \end{aligned}\right\} \Rightarrow \frac{dx}{dt}(0) = \frac{KA - x(0)}{T} = \frac{-d}{T}$$

A straight line through the point **[0,x(0)]** with this gradient with also go through the point

[T, x(0)-d] = [T,KA]

Add this line to the plot before doing the sketch. [GET TO STEADY-STATE IN ONE TIME CONSTANT]

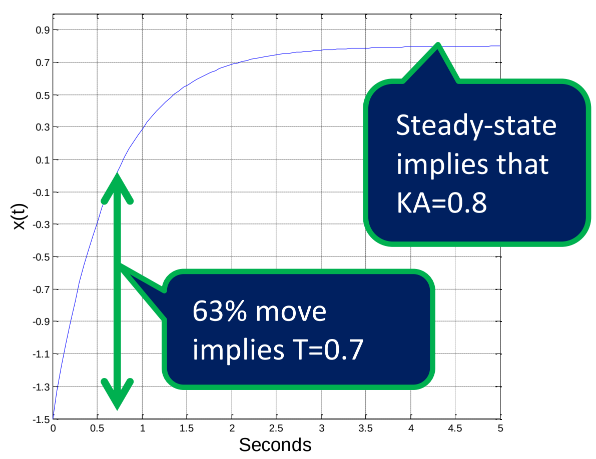
EXAMPLE 1 *(With initial gradient line added)*



REVERSE COMPUTATIONS: Estimate time constant T and gain K from the step response for input of magnitude A using the two observations:

1) Steady-state is KA and 2) time to move 63% of way to steady-state is T (or 37% from KA).

- Total move is -1.5 ➔0.8 or 2.3 up.
- x(T) = 0.9 - 0.37*2.3 = -0.05



- Total move is 25 ➔5 or 20 down.
- x(T) = 5 - 0.37*(-20) = 12.4