

# Modelling and control summaries



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## 1<sup>st</sup> order responses 6: Using MATLAB

Assume a 1st order model with constant coefficients, output  $x(t)$  and constant input  $u(t)$ .

$$a \frac{dx}{dt} + bx = cu(t), \quad x(0) = x_0, \quad u(t) = A, t \geq 0$$

### ANALYTIC SOLUTIONS USING DSOLVE.M

MATLAB will give an exact analytic solution.

Here some screen dumps are used to illustrate the format. Use 'help dsolve' to find out more.

Take the ODE and initial condition

$$6 \frac{dx}{dt} + 4x = 3; \quad x(0) = 2$$

Note how these parameters are entered directly into the dsolve.m command.

A common mistake is to write, for example, '4x' instead of '4\*x'. MATLAB is a computer code so you must be explicit about the need for multiplication.

The solution is a symbolic variable so you need to understand how to use symbolic variables in MATLAB

```
MATLAB R2014a
>> xt=dsolve('6*Dx+4*x=3','x(0)=2')
xt =
(5*exp(-(2*t)/3))/4 + 3/4
```

MATLAB is flexible and allows you to use what ever variables you like.

$$0.1 \frac{dy}{dt} - 2.1y = 0.5; \quad y(0) = -1$$

```
MATLAB R2014a
>> yt=dsolve('0.1*Dy-2.1*y=0.5','y(0)=-1')
yt =
-(16*exp(21*t))/21 - 5/21
```

## NUMERICAL SOLUTIONS WITH MATLAB VIA DSOLVE

A symbolic expression can be evaluated using subs.m

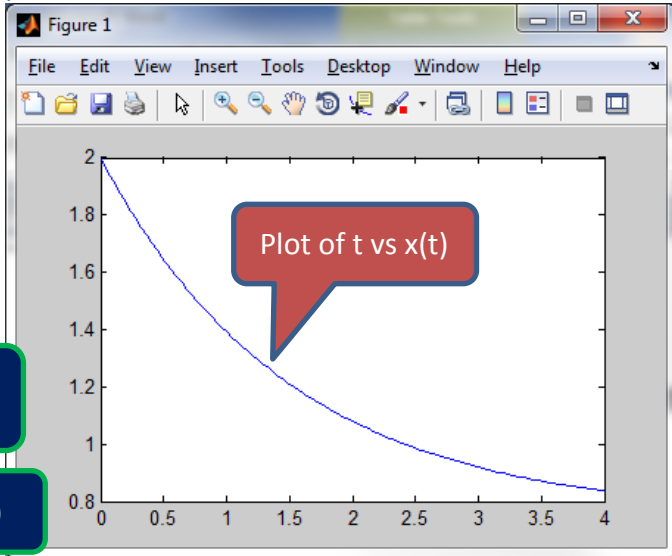
Once numerical values are known, the function can be plotted.

```

MATLAB R2014a
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x_t =
(5*exp(-(2*t)/3))/4 + 3/4
>> t=linspace(0,4,100);
>> x_at_t=subs(x_t,t);
fx >> plot(t,x_at_t)
    
```

Define domain

Solve for x(t)



## NUMERICAL SOLUTIONS WITH MATLAB VIA LAPLACE TOOLS

The key m-file is tf.m which forms a Laplace transform (or transfer function).

Hence, first the student must form the Laplace transform and then enter this using tf.m

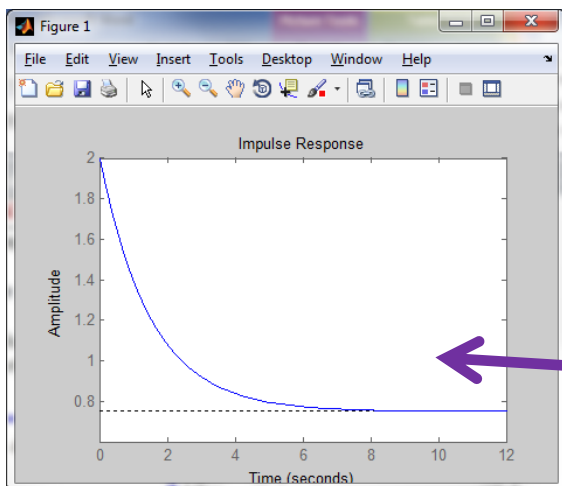
Inverse Laplace in terms of numeric values is formed using impulse.m which can produce a figure. To obtain values in workspace, use 'help impulse' to find out more.

$$6 \frac{dx}{dt} + 4x = 3; \quad x(0) = 2 \Rightarrow (6s + 4)X(s) - 6x(0) = \frac{3}{s}$$

$$\Rightarrow X(s) = \frac{3 + 6sx(0)}{s(6s + 4)} = \frac{3 + 12s}{6s^2 + 4s}$$

```

MATLAB R2014a
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>> X=tf([12,3],[6 4 0])
X =
12 s + 3
-----
6 s^2 + 4 s
fx
    
```



```

MATLAB R2014a
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>>
fx >> impulse(X)
    
```