Modelling and control summaries

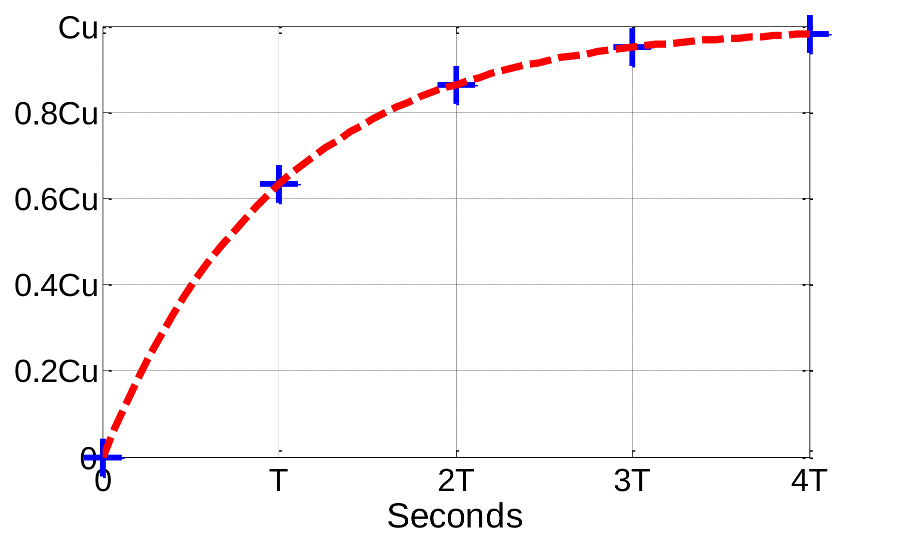
by Anthony Rossiter

1^{st} order responses 9: parameter dependence

Assume a a 1st order model with constant coefficients, output x(t) and constant input u(t).	The response is known:
$T\dfrac{dx}{dt}+x=Cu$	$x(t)=Cu+[x(0)-Cu]e^{-\frac{t}{T}}$

Explore how the response changes as the parameters of a model change – here a simple step response is given.

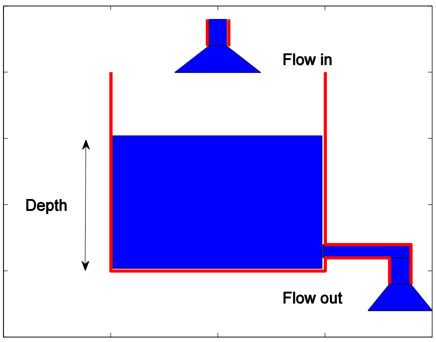
Note that the decay rate is linked directly to 'T' and the steady-state to Cu.



SIMPLE TANK MODEL

A tank level system with input flow f_{in} and depth h is governed by the following model (g, ρ are gravity and liquid density). Show how the behaviour changes as:

1. The cross-sectional area A varies.
2. The resistance R of the outflow pipe varies.



$$A\frac{dh}{dt}+\frac{\rho g}{R}h=f_{in}$$

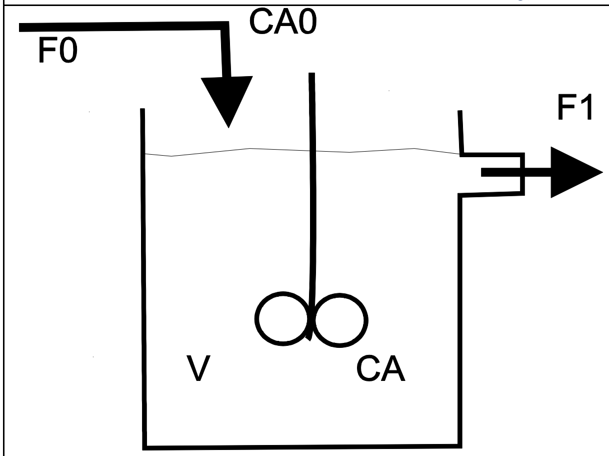
First change into time constant form:

$$\underbrace{\left(\frac{AR}{\rho g}\right)}_{T}\frac{dh}{dt}+h=\underbrace{\left(\frac{R}{\rho g}\right)}_{C}f_{in}$$

1. As A increases, T increases so system response slows but C is unaffected so the steady-state is unaffected.
2. As R increases, T increases so system slows and C increases so steady-state depth increases.

MIXING TANK

Volume V, inlet concentration C_{A0}, outlet concentration C_A, flow rate F_0.



The model in time constant form is:

$$\frac{V}{F_0}\frac{dC_A}{dt} + C_A = C_{A0}$$

Hence:

1. As V increases, T increases and the system response slows.
2. As F_0 increases, T decreases and the system response speeds up.
3. Neither V or F affect the steady-state.

HEATING SYSTEM

Input power U, thermal capacitance C and heat loss coefficient to the surroundings k

$$U = C\frac{d\theta}{dt} + k(\theta - \theta_o)$$

$$\underbrace{\left(\frac{C}{k}\right)}_{T}\frac{d\theta}{dt} + \theta = \theta_o + \frac{1}{k}U$$

For convenience, consider deviation from ambient temperature θ_0 only:

1. Time constant is C/k, so slows as C increases but speeds up as k increases.
2. Gain is (1/k) so steady-state increases as k decreases.

REMARKS:

The conclusions given above are largely common sense, that is exactly what you would expect from considering the expected behaviour without recourse to mathematics and formal modelling. It is good practice to think about what you expect using your knowledge of the world and check the mathematics agree with this.

TUTORIAL

Carry out an equivalent analysis for other common first order models and determine the behaviour dependence on different parameters.

1. RL and RC electrical circuits.
2. Mass damper and spring-damper systems.
3. Climb rate of an aircraft.
4. Pulley-damper.

Conjecture how you would design a system to give the desired behaviour, for example how would you design a tank system which had a specified time constant and gain and stored at least 1000 gallons?