Modelling and control summaries

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2nd order responses 2: over-damped systems

Assume a standard 2nd order model with constant coefficients, output y(t) and input u(t). What is the expected behaviour and how does this vary with a,b,c?	$a\frac{d^2x}{dt^2}+b\frac{dx}{dt}+cx=f$
For now the focus is on constant f, usually f=0 or f=1 and zero initial conditions (x(0)=0, dx/dt(0)=0)	

An overdamped system is where the characteristic polynomial marked below has real roots

$$a\frac{d^2x}{dt^2}+b\frac{dx}{dt}+cx=f \quad\Rightarrow\quad ap^2+bp+c=0 \quad\Rightarrow\quad p=p_1,p_2$$

The previous note derived a general solution. The aim here is consider the impact of large differences in the values for p_1 and p_2.	$x=\frac{f}{c}\left[1+\left(\frac{1}{p_2-p_1}\right)(p_1e^{p_2t}-p_2e^{p_1t})\right]$

COMPARE COEFFICIENTS

Compare the coefficients for each exponential in the solution. **They have a magnitude ratio of $p_1:p_2$**	$C_1=\frac{f}{c}\frac{-p_2}{(p_2-p_1)};\quad C_2=\frac{f}{c}\frac{p_1}{(p_2-p_1)}$ If $\lvert p_1\rvert >> \lvert p_2\rvert$, then $\lvert C_2\rvert >> \lvert C_1\rvert$, so the slower mode will dominate.

EXAMPLE 1

$$\ddot{x}+10\dot{x}+9x=1$$

$$p^2+10p+9=0$$

$$\Rightarrow \quad p_1=-1,\ p_2=-9$$

$$x=\frac{1}{c}\left[1+\left(\frac{1}{p_2-p_1}\right)(p_1e^{p_2t}-p_2e^{p_1t})\right]$$

$$x=\frac{1}{9}\left[1+\left(\frac{1}{-8}\right)(-e^{-9t}+9e^{-t})\right]$$

$$x=\frac{1}{9}+\left(\frac{1}{72}\right)(e^{-9t}-9e^{-t})$$

Slower mode is dominating

EXAMPLE 2

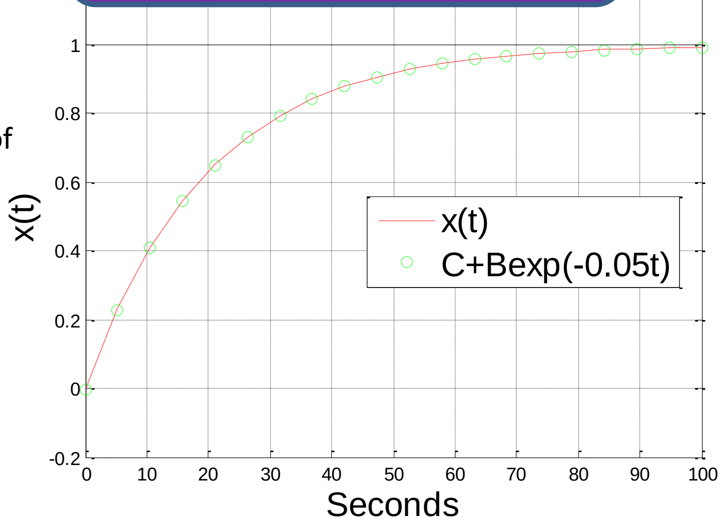
$$\ddot{x} + 20\dot{x} + x = 1$$
$$p_1 = -19.95,\ p_2 = -0.05$$

$$x = \left[1 + \left(\frac{1}{-19.9}\right)(-0.05e^{-19.95t} + 19.95e^{-0.05t})\right]$$
$$x = 1 + 0.0025e^{-19.95t} - 1.0025e^{-0.05t})$$

Note that the fast pole has a very small residue, as well as converging much faster.

A comparison of the response with and without the faster mode is interesting.

From a visual perspective the plots are indistinguishable and thus the response of this overdamped 2nd order system is, in essence, equivalent to a 1st order system with just the slower pole.



SUMMARY

Heavily over-damped 2nd order systems can often be approximated by a 1st order system with just the slower dynamic.

Note that here we assumed no system zeros and zero initial conditions.

Examples for students to try

(Check answers with MATLAB)

$$\ddot{x} + 40\dot{x} + 76x = 1; \quad \ddot{x} + 15\dot{x} + 14x = 3$$
$$roots\ -2, -38 \qquad -14, -1$$