Modelling and control summaries

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2nd order responses 7: steady-state forms

Assume a standard 2nd order model with constant coefficients, output x(t) and input u(t). What is the expected behaviour and how does this vary with damping ratio?	$\frac{d^2x}{dt^2}+2\varsigma w_n\frac{dx}{dt}+w_n^2x=f$

This resource focuses on the expected steady-state: $\lim_{t\to\infty} x(t)$

It is assumed that the steady-state exists and thus the damping ratio is positive.

Definition of steady-state

At steady-state, x(t) is constant and therefore:

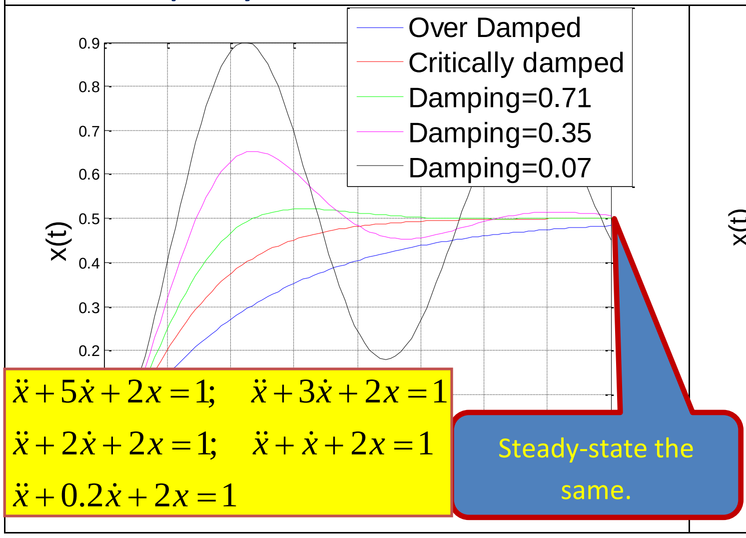
$$\lim_{t\to\infty}\frac{dx}{dt}=0;\quad \lim_{t\to\infty}\frac{d^2x}{dt^2}=0;$$

Substitute these into the differential equation and hence:

$$\left\{\frac{d^2x}{dt^2}+2\varsigma w_n\frac{dx}{dt}+w_n^2x=f\right\}\quad AND\quad \left\{\frac{dx}{dt}=0,\quad \frac{d^2x}{dt^2}=0\right\}\quad\Rightarrow\quad w_n^2x=f\quad OR\quad x=\frac{f}{w_n^2}$$

SUMMARY: The asymptotic value of x depends solely on the natural frequency and the input signal.

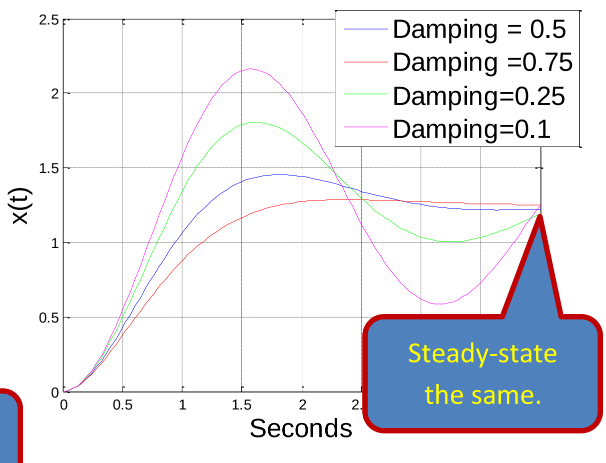
Illustrations of steady-state being unaffected by changes in the damping ratio but where *f* and natural frequency w_n are fixed.



$\ddot{x}+5\dot{x}+2x=1;\quad \ddot{x}+3\dot{x}+2x=1$

$\ddot{x}+2\dot{x}+2x=1;\quad \ddot{x}+\dot{x}+2x=1$

$\ddot{x}+0.2\dot{x}+2x=1$



EXAMPLES

Find the steady-state for the following systems/input pairs (you can assume convergence).

$$\ddot{x} + 5\dot{x} + 10x = 1 \quad \Rightarrow \quad \lim_{t\to\infty} x(t) = \frac{1}{10}$$

$$\ddot{x} + 3\dot{x} + 4x = 2 \quad \Rightarrow \quad \lim_{t\to\infty} x(t) = \frac{2}{4}$$

$$\ddot{x} + 2\dot{x} + 6x = 0.4 \quad \Rightarrow \quad \lim_{t\to\infty} x(t) = \frac{0.4}{6}$$

$$\ddot{x} + \dot{x} + 5x = 1.2 \quad \Rightarrow \quad \lim_{t\to\infty} x(t) = \frac{1.2}{5}$$

$$\ddot{x} + 0.2\dot{x} + 10x = 3 \quad \Rightarrow \quad \lim_{t\to\infty} x(t) = \frac{3}{10}$$

SUMMARY

1. Steady-state is not affected by changes in damping alone.
2. Steady-state can be computed quickly by setting all the derivatives to zero.