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State-space models 1 Introduction

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Introduction

- These resources look at a state-space models, their origins, properties and use in control.
- The first few resources focus on origins.

1. What is a state space model?
2. How do I derive a model to represent a given system?
3. Linearised models.

Viewers may find it useful to review the sections on 1st and 2nd order modelling.

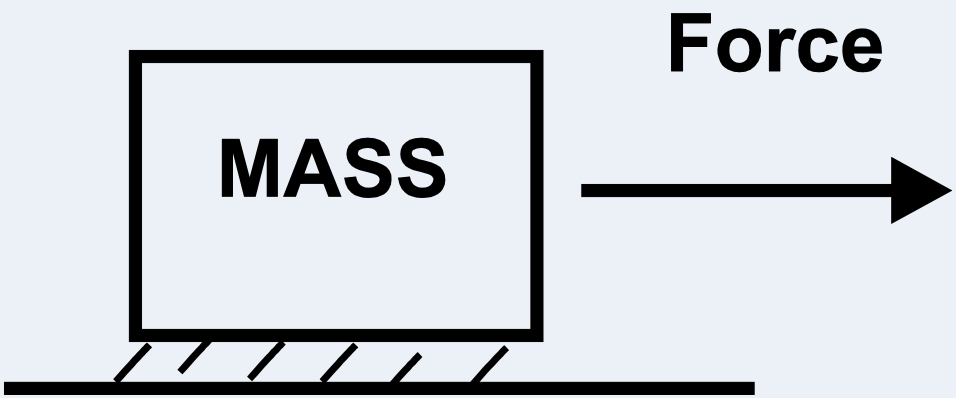
State space models

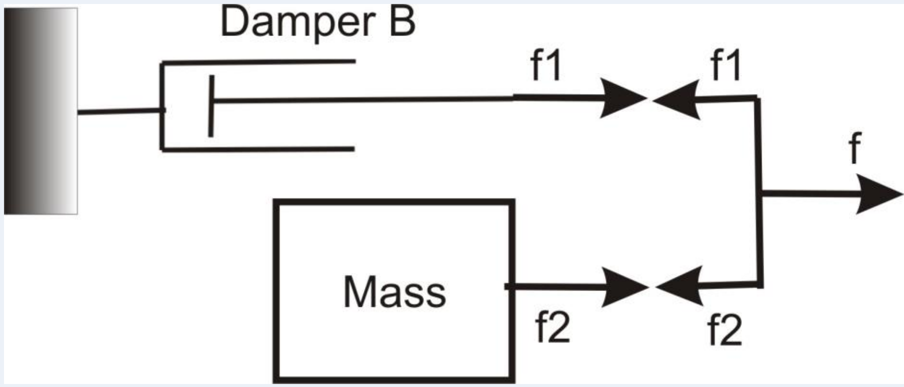
- State space models are defined in terms of so called system states.
- States are properties which change with time such as speed, temperature, pressure and so forth.
- The model is defined in terms of the derivatives of the states. If you know the derivatives of all the states, then you can capture the system behaviour.

Mass-damper

A simple example of a mass-damper could be a vehicle.

The engine provides a drive force whereas, for example, wind and road provide friction or drag.

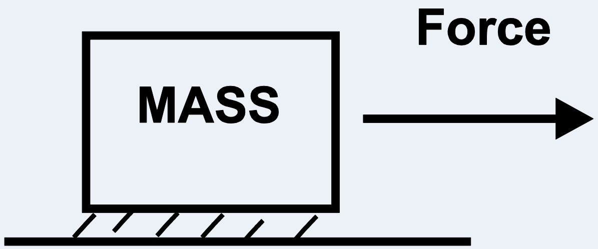


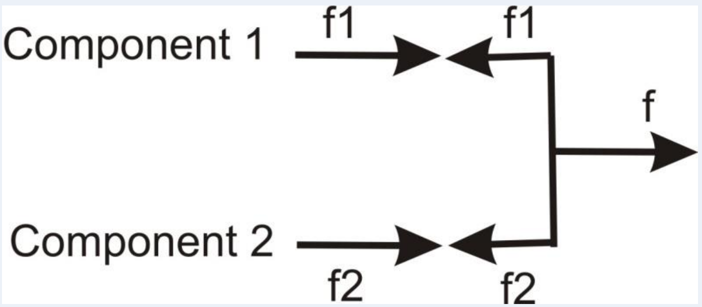


How do we model such a scenario? (**Slower discussion available in modelling sections.**)

Modelling a mass-damper

Force balance can now be used to determine the overall model of behaviour.





$$\left.\begin{aligned} f1 &= \hat{B}v \\ f2 &= M\frac{dv}{dt} \\ f &= f1 + f2 \end{aligned}\right\} \Rightarrow$$

This form of mass-damper gives a simple first order differential equation.

Defining state derivatives

The mass-damper can be thought of as having a single state, that is the velocity.

A state space model re-arranges the equation to define the derivative of the state.

$$M\frac{dv}{dt}+\hat{B}v=f \quad \Rightarrow \quad \frac{dv}{dt}=-\frac{\hat{B}}{M}v+\frac{1}{M}f$$

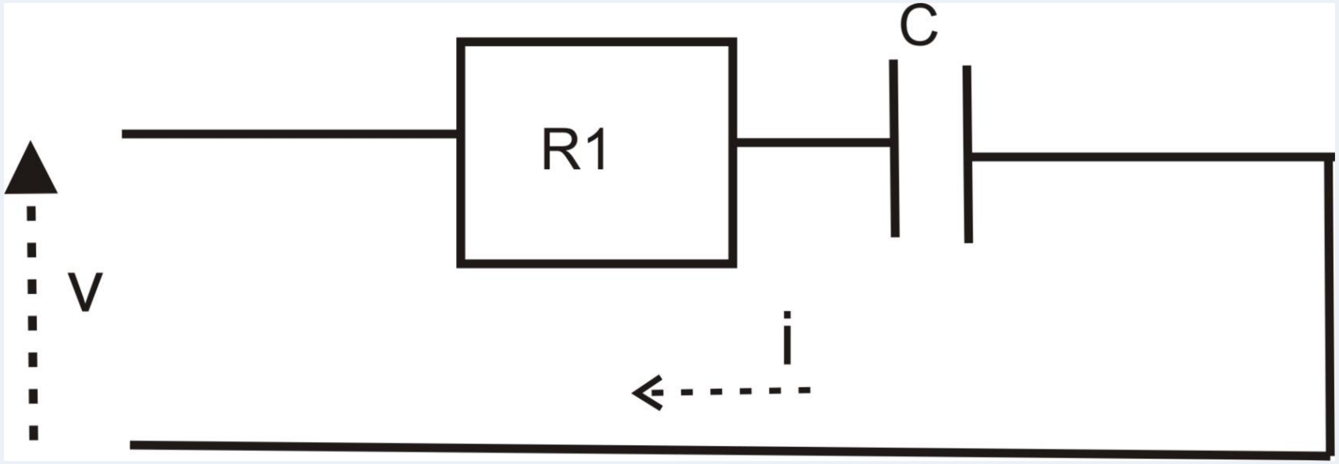
This derivative is expressed with matrix notation

$$\frac{dv}{dt}=\underbrace{\left[-\frac{\hat{B}}{M}\right]}_{A}v+\underbrace{\left[\frac{1}{M}\right]}_{B}f \quad \Rightarrow \quad \dot{v}=Av+Bf$$

State derivative is linear in the state and the input.

Resistor-capacitor in series

Consider the following circuit and use Kirchhoff's voltage law to derive an appropriate model.



$$\left.\begin{aligned} v1 = iR1 = R1\frac{dq}{dt} \\ v2 = \frac{1}{C}q \\ v = v1 + v2 \end{aligned}\right\} \Rightarrow \quad R1\frac{dq}{dt} + = v$$

Defining state derivatives

The resistor-capacitor can be thought of as having a single state, that is the charge across the capacitor.

A state space model re-arranges the equation to define the derivative of the state.

$$R\frac{dq}{dt}+\frac{1}{C}q=v \quad \Rightarrow \quad \frac{dq}{dt}=-\frac{1}{RC}q+\frac{1}{R}v$$

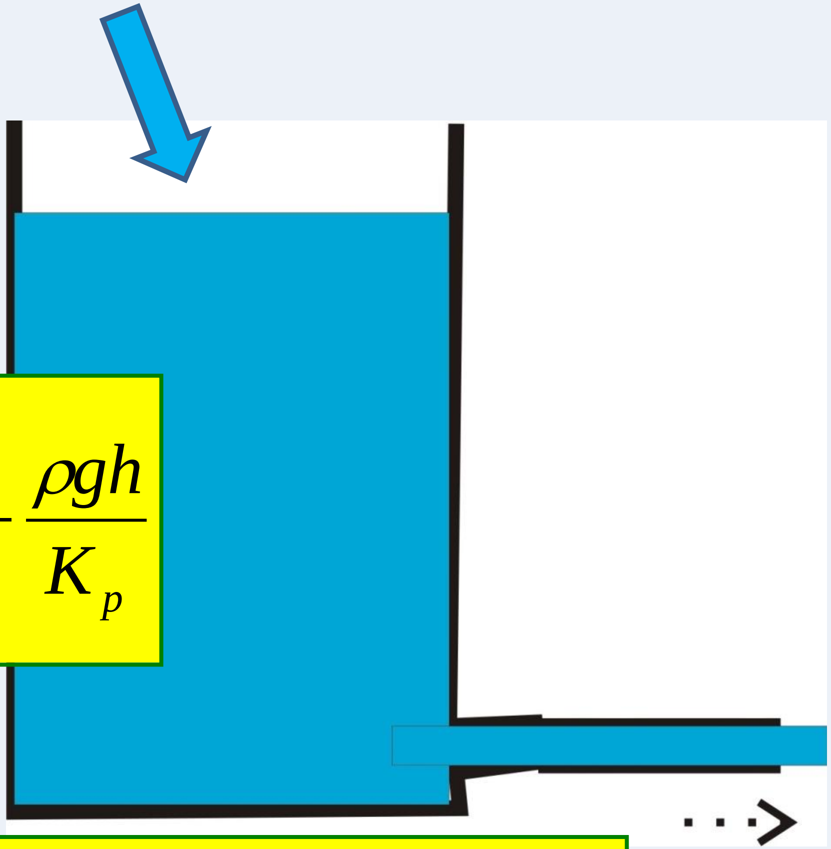
This derivative is expressed with matrix notation

$$\frac{dq}{dt}=\underbrace{\left[-\frac{1}{RC}\right]}_{A}q+\underbrace{\left[\frac{1}{R}\right]}_{B}v \quad \Rightarrow \quad \dot{q}=Aq+Bv$$

State derivative is linear in the state and the input.

Tank with outlet and some inflow

Consider an inflow F_{in} and update the model for the change in depth.



$$\left.\begin{aligned} \hat{A}\frac{dh}{dt} &= F_{in} - F_{out} \\ K_p F_{out} &= \rho g h \end{aligned}\right\} \Rightarrow \quad \hat{A}\frac{dh}{dt} = F_{in} - \frac{\rho g h}{K_p}$$

$$\frac{dh}{dt} = \underbrace{\left[-\frac{\rho g}{\hat{A}K_p} \right]}_{A} h + \underbrace{\left[\frac{1}{\hat{A}} \right]}_{B} F_{in} \quad \Rightarrow \quad \dot{h} = Ah + BF_{in}$$

State derivative is linear in the state and the input.

Summary

Illustrated state-space model derivation for systems that can be modelling a simple 1st order ODE.

This amounts to a minor rearrangement and expressing the model as an equation which defines the state derivative with **linear dependence on a state and an input**.

$$T\frac{dx}{dt} + x = Ku \quad \Rightarrow \quad \dot{x} = \underbrace{\left[-\frac{1}{T}\right]}_{A} x + \underbrace{\left[\frac{K}{T}\right]}_{B} u$$

$$\dot{x} = Ax + Bu$$

It is common to use matrix names A,B for the coefficients of the state and input respectively.

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