



State-space models 1

Introduction

J A Rossiter

Introduction

- These resources look at a state-space models, their origins, properties and use in control.
- The first few resources focus on origins.
 1. What is a state space model?
 2. How do I derive a model to represent a given system?
 3. Linearised models.

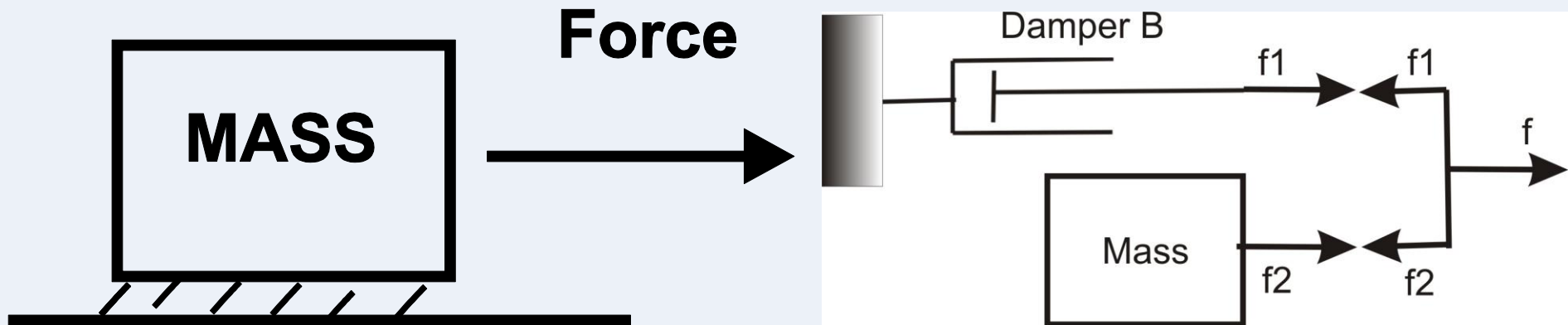
Viewers may find it useful to review the sections on 1st and 2nd order modelling.

State space models

- State space models are defined in terms of so called system states.
- States are properties which change with time such as speed, temperature, pressure and so forth.
- The model is defined in terms of the derivatives of the states. If you know the derivatives of all the states, then you can capture the system behaviour.

Mass-damper

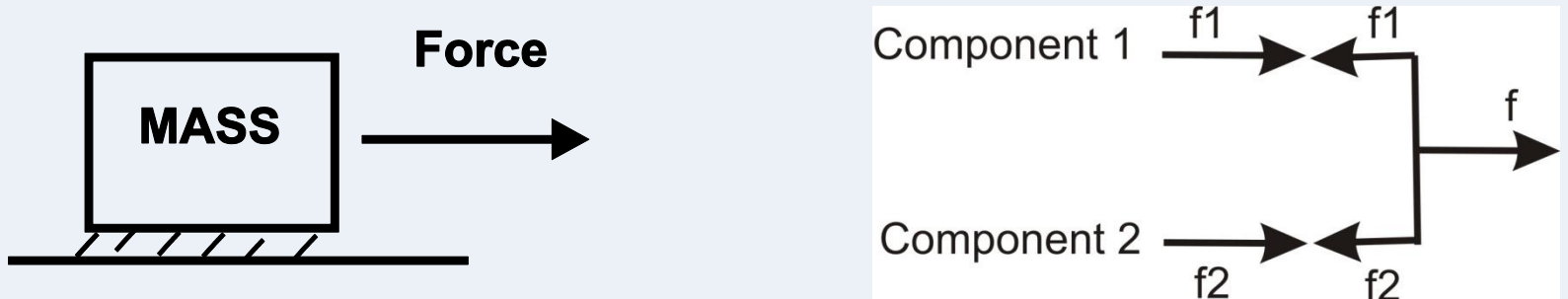
A simple example of a mass-damper could be a vehicle. The engine provides a drive force whereas, for example, wind and road provide friction or drag.



How do we model such a scenario? (**Slower discussion available in modelling sections.**)

Modelling a mass-damper

Force balance can now be used to determine the overall model of behaviour.



$$\left. \begin{aligned} f_1 &= \hat{B}v \\ f_2 &= M \frac{dv}{dt} \\ f &= f_1 + f_2 \end{aligned} \right\} \Rightarrow$$



This form of mass-damper gives a simple first order differential equation.

Defining state derivatives

The mass-damper can be thought of as having a single state, that is the velocity.

A state space model re-arranges the equation to define the derivative of the state.

$$M \frac{dv}{dt} + \hat{B}v = f \quad \Rightarrow \quad \frac{dv}{dt} = -\frac{\hat{B}}{M}v + \frac{1}{M}f$$

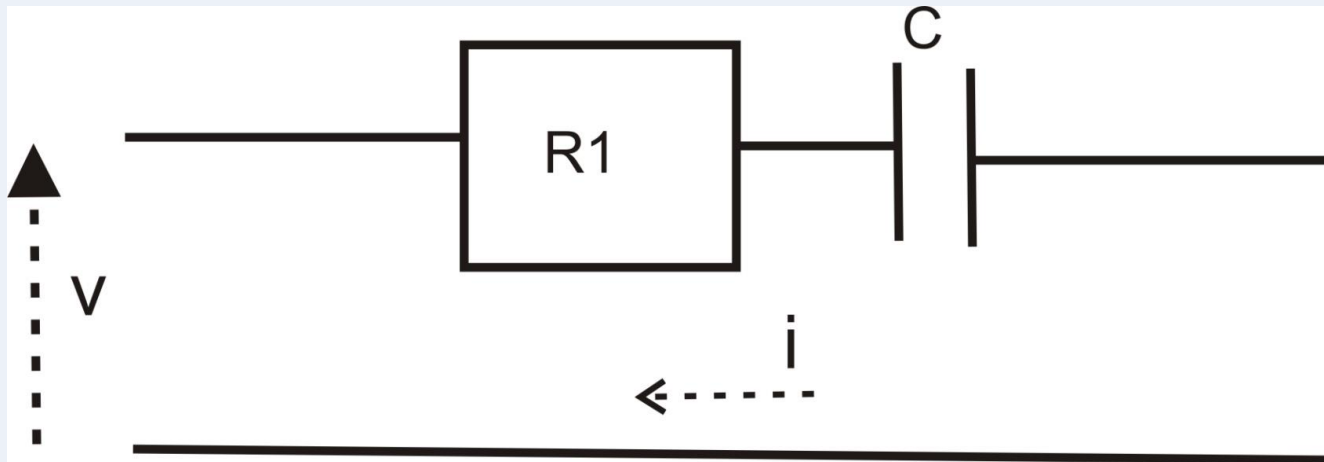
This derivative is expressed with matrix notation

$$\frac{dv}{dt} = \underbrace{\begin{bmatrix} -\frac{\hat{B}}{M} \end{bmatrix}}_A v + \underbrace{\begin{bmatrix} \frac{1}{M} \end{bmatrix}}_B f \quad \Rightarrow \quad \dot{v} = Av + Bf$$

State derivative is linear
in the state and the input.

Resistor-capacitor in series

Consider the following circuit and use Kirchhoff's voltage law to derive an appropriate model.



$$\left. \begin{aligned} v_1 &= iR_1 = R_1 \frac{dq}{dt} \\ v_2 &= \frac{1}{C} q \\ v &= v_1 + v_2 \end{aligned} \right\} \Rightarrow R_1 \frac{dq}{dt} + \frac{q}{C} = v$$

Defining state derivatives

The resistor-capacitor can be thought of as having a single state, that is the charge across the capacitor.

A state space model re-arranges the equation to define the derivative of the state.

$$R \frac{dq}{dt} + \frac{1}{C} q = v \quad \Rightarrow \quad \frac{dq}{dt} = -\frac{1}{RC} q + \frac{1}{R} v$$

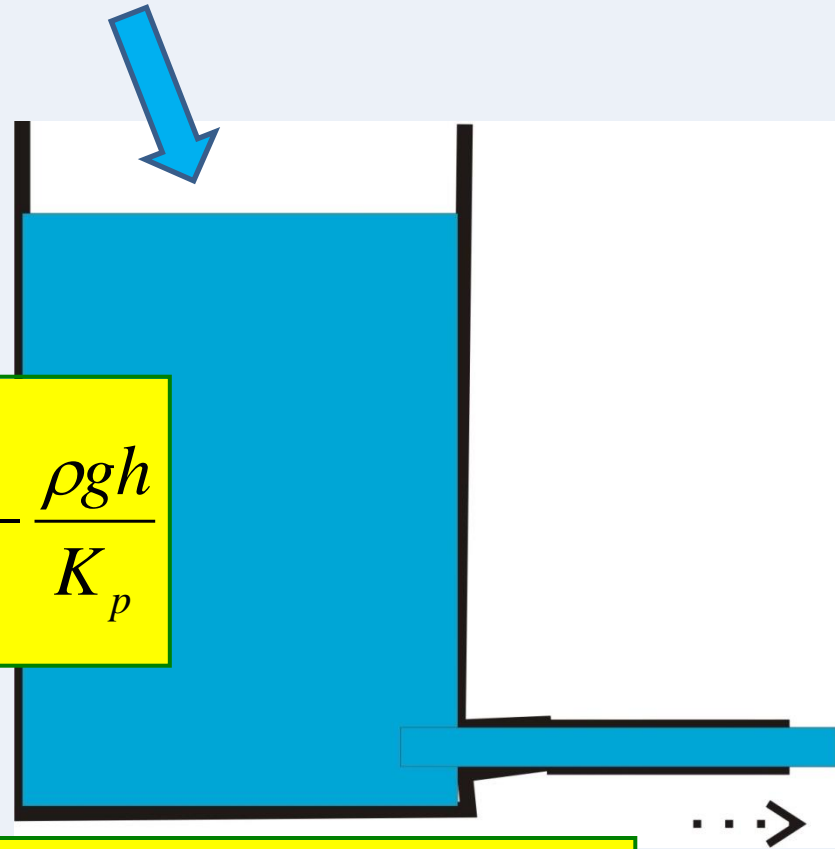
This derivative is expressed with matrix notation

$$\frac{dq}{dt} = \underbrace{\begin{bmatrix} -\frac{1}{RC} \end{bmatrix}}_A q + \underbrace{\begin{bmatrix} \frac{1}{R} \end{bmatrix}}_B v \quad \Rightarrow \quad \dot{q} = Aq + Bv$$

State derivative is linear in the state and the input.

Tank with outlet and some inflow

Consider an inflow F_{in} and update the model for the change in depth.



$$\left. \begin{aligned} \hat{A} \frac{dh}{dt} &= F_{in} - F_{out} \\ K_p F_{out} &= \rho g h \end{aligned} \right\} \Rightarrow \hat{A} \frac{dh}{dt} = F_{in} - \frac{\rho g h}{K_p}$$

$$\frac{dh}{dt} = \underbrace{\left[-\frac{\rho g}{\hat{A} K_p} \right]}_A h + \underbrace{\left[\frac{1}{\hat{A}} \right]}_B F_{in} \Rightarrow \dot{h} = Ah + BF_{in}$$

State derivative is linear in the state and the input.

Summary

Illustrated state-space model derivation for systems that can be modelling a simple 1st order ODE.

This amounts to a minor rearrangement and expressing the model as an equation which defines the state derivative with **linear dependence on a state and an input**.

$$T \frac{dx}{dt} + x = Ku \quad \Rightarrow \quad \dot{x} = \underbrace{\begin{bmatrix} -\frac{1}{T} \end{bmatrix}}_A x + \underbrace{\begin{bmatrix} \frac{K}{T} \end{bmatrix}}_B u$$
$$\dot{x} = Ax + Bu$$

It is common to use matrix names A,B for the coefficients of the state and input respectively.



Anthony Rossiter
Department of Automatic Control and
Systems Engineering
University of Sheffield
www.shef.ac.uk/acse

© 2016 University of Sheffield

This work is licensed under the Creative Commons Attribution 2.0 UK: England & Wales Licence. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/2.0/uk/> or send a letter to: Creative Commons, 171 Second Street, Suite 300, San Francisco, California 94105, USA.



It should be noted that some of the materials contained within this resource are subject to third party rights and any copyright notices must remain with these materials in the event of reuse or repurposing.

If there are third party images within the resource please do not remove or alter any of the copyright notices or website details shown below the image.

(Please list details of the third party rights contained within this work.

If you include your institutions logo on the cover please include reference to the fact that it is a trade mark and all copyright in that image is reserved.)