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State-space models 11 tutorial sheet

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Introduction

The prime role of this video is to present a number of worked examples for students to practice.

You can pause the video and try the problems for yourself before looking at the solutions.

You can use MATLAB to test your answers.

Reminder of a key result

$$Y(s) = \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0} U(s)$$

$$\frac{d}{dt}[z] = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_n \end{bmatrix}}_{A} z + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}}_{B} u;$$

$$y = \underbrace{\begin{bmatrix} b_0 & b_1 & \cdots & b_{n-1} \end{bmatrix}}_{C} z$$

NOTE: States can be in reverse order with obvious impact on matrix definitions!

Find a state space model in controllable canonical form

$$7\frac{du}{dt} - 3u = \frac{d^5x}{dt^5} + 2\frac{d^4x}{dt^4} - 2\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 4x$$

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Find a state space model in controllable canonical form

$$Y(s) = \frac{s^3 + 4s - 1}{s^4 + s^3 + s} U(s)$$

Find a state space model in diagonal canonical form

$$Y(s) = \frac{4s+1}{s^3+7s^3+14s+8}U(s)$$

Find a state space model in controllable canonical form

$$Y(z)=\frac{z^2-z+0.3}{z^4+1.2z^3-0.6z^2+0.2}U(z)$$

Find a transfer function equivalent for the following state space model

$$\frac{d}{dt}[x] = \underbrace{\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 8 \\ -1 \end{bmatrix}}_{B} f; \quad y = \begin{bmatrix} 1 & 6 \end{bmatrix} x$$

Find a transfer function equivalent for the following state space model

$$\frac{d}{dt}[z] = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 4 & 2 \end{bmatrix}}_{A} z + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u; \quad y = \underbrace{\begin{bmatrix} 1 & -3 & 2 & 5 \end{bmatrix}}_{C} z$$

For the state space model, find a model with states defined by the transformation z=Tx.

$$\frac{d}{dt}[x] = \underbrace{\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 8 \\ -1 \end{bmatrix}}_{B} f; \quad y = [1 \quad 6]x; \quad T = \begin{bmatrix} 0.5 & 0.866 \\ -0.866 & 0.5 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0.5 & -0.866 \\ 0.866 & 0.5 \end{bmatrix}$$

$$TT^{-1}\dot{z} = \underbrace{TAT^{-1}}_{\hat{A}} z + \underbrace{TB}_{\hat{B}} u$$

$$y = \underbrace{CT^{-1}}_{\hat{C}} z$$

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