State-space feedback 1 introduction

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Introduction

- The first three sections looked at the definition of state space models, the computation of underlying behaviours and concepts of controllability and observability.

$$\dot{x} = Ax + Bu; \quad y = Cx + Du$$

- The next job is to look at feedback design, that is how do we embed feedback into a state space system in order to obtain the desired behaviour.
- This series assumes that state measurements are available for use by the control law.

What is state feedback?

State feedback means that state measurements can be used to determine the control action.

For simplicity, for now, we consider the regulation case which means the target is the origin.

State feedback means the following.

$$u = -Kx$$

What impact does such a control law have on behaviour?

Impact of state feedback on behaviour

It is obvious that feedback changes behaviour.

This is well known thus discussed not further.

In the state feedback case the following analysis is straightforward.

$$\left\{ \begin{array}{c} \dot{x} = Ax + Bu \\ u = -Kx \end{array} \right\} \quad \Rightarrow \quad \dot{x} = \underbrace{(A - BK)}_{\Phi} x$$

Closed-loop behaviour is governed by the eigenvalues of the matrix Φ=A-BK.

Controllability

It is clear that state feedback changes behaviour and allows us to move the closed-loop poles.

OPEN-LOOP

$$\dot{x} = Ax + Bu$$

Poles are eigenvalues of A

CLOSED-LOOP

$$\dot{x} = \underbrace{(A - BK)}_{\Phi} x$$

Poles are eigenvalues of A-BK

Controllability (discrete case)

IDENTICAL IN PRINCIPLE TO CONTINUOUS CASE – NOT DISCUSSED FURTHER.

OPEN-LOOP

$$x_{k+1} = Ax_k + Bu_k$$

Poles are eigenvalues of A

CLOSED-LOOP

$$x_{k+1} = \underbrace{(A - BK)}_{\Phi} x_k$$

Poles are eigenvalues of A-BK

Controllability questions

While it is clear that K allows us to move the poles:

- Is it possible to place the poles arbitrarily?
- What is a systematic design method for selecting K?

Some examples will show that selection of K is non-trivial in general.

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Find the impact of the state feedback

$$\dot{x} = Ax + Bu; \quad A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}; \quad K = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Determine the eigenvalues with and without the state feedback.

OPEN-LOOP

$$|\lambda I - A| = 0 \quad \Rightarrow \quad \lambda = 2, -1$$

CLOSED-LOOP

$$BK = \begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix}; \quad A - BK = \begin{bmatrix} 3 & 6 \\ -2 & -6 \end{bmatrix};$$

$$|\lambda I - (A - BK)| = \begin{vmatrix} \lambda - 3 & -6 \\ 2 & \lambda + 6 \end{vmatrix} \quad \Rightarrow \quad \lambda = 1.37, -4.27$$

Find the dependence of the poles on the state feedback

$$\dot{x} = Ax + Bu; \quad A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}; \quad K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

$$BK = \begin{bmatrix} -2k_1 & -2k_2 \\ 3k_1 & 3k_2 \end{bmatrix}; \quad A - BK = \begin{bmatrix} 1+2k_1 & 2+2k_2 \\ 1-3k_1 & -3k_2 \end{bmatrix}$$

$$\left|\lambda I - (A - BK)\right| = \lambda^2 - (1 + 2k_1 - 3k_2)\lambda - (1 + 2k_1)3k_2 - (2 + 2k_2)(1 - 3k_1)$$

$$\left|\lambda I - (A - BK)\right| = \lambda^2 - (1 + 2k_1 - 3k_2)\lambda - 2 + 6k_1 - 5k_2$$

In principle we can now choose parameters to get desired poles, but this is cumbersome.

Find pole dependence on feedback

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}; \quad K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

$$BK = \begin{bmatrix} k_1 & k_2 & k_3 \\ 2k_1 & 2k_2 & 2k_3 \\ -3k_1 & -3k_2 & -3k_3 \end{bmatrix}; \quad A - BK = \begin{bmatrix} 2-k_1 & 1-k_2 & 3-k_3 \\ 3-2k_1 & 1-2k_2 & 4-2k_3 \\ 1+3k_1 & 3k_2 & 1+3k_3 \end{bmatrix}$$

$$\left|\lambda I - (A - BK)\right| = \lambda^3 + p_2\lambda^2 + p_1\lambda + p_0$$

Not paper and pen computations in general.
We need better design tools!

Summary

Introduced concepts of state feedback.

1. Easy to postulate a feedback mechanism where the input depends upon state measurements.

$$u = -Kx$$

2. Easy to show that such a feedback changes the pole positions.
3. Less clear cut whether a brute force computation is an effective mechanism for choosing the parameters of K to achieve the desired poles.

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