State-space feedback 6 challenges of pole placement

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Introduction

- The earlier videos introduced the concept of state feedback and demonstrated that it moves the poles.

$$\left\{\begin{matrix} \dot{x} = Ax + Bu \\ u = -Kx \end{matrix}\right\} \quad \Rightarrow \quad \dot{x} = \underbrace{(A - BK)}_{\Phi}x$$

- It was shown that when a system is fully controllable, the poles can be placed arbitrarily, that is wherever the user desires.
- This video considers the repercussions of having to place all the poles – so called POLE PLACEMENT.
- **Discrete time case uses same concepts/algebra.**

Pole placement with canonical forms

One can form the closed-loop state space model by inspection.

$$\left\{\begin{matrix} \dot{x} = Ax + Bu \\ u = -Kx \end{matrix}\right\} \Rightarrow \quad \dot{x} = \underbrace{(A - BK)}_{\Phi} x$$

$$A - BK = \begin{bmatrix} -a_{n-1} - k_1 & -a_{n-2} - k_2 & \cdots & -a_1 - k_{n-1} & -a_0 - k_n \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

$$|\lambda I - A + BK| = \lambda^n + (a_{n-1} + k_1)\lambda^{n-1} + \cdots + (a_0 + k_n)$$

One can choose the parameters of the closed-loop pole polynomial directly by choosing the parameters k_i.

Behaviours

This video will look at the consequences of pole placement.

1. How easily can one determine a good location for each and every pole.
2. What if the target locations are poorly chosen.
3. Can one come up with a systematic design methodology.

It will be shown that being able to place the poles is not the same as being able to place the poles well.

NUMERICAL EXAMPLES

Example 1

Compare the closed-loop behaviour with different choices of outputs and target poles.

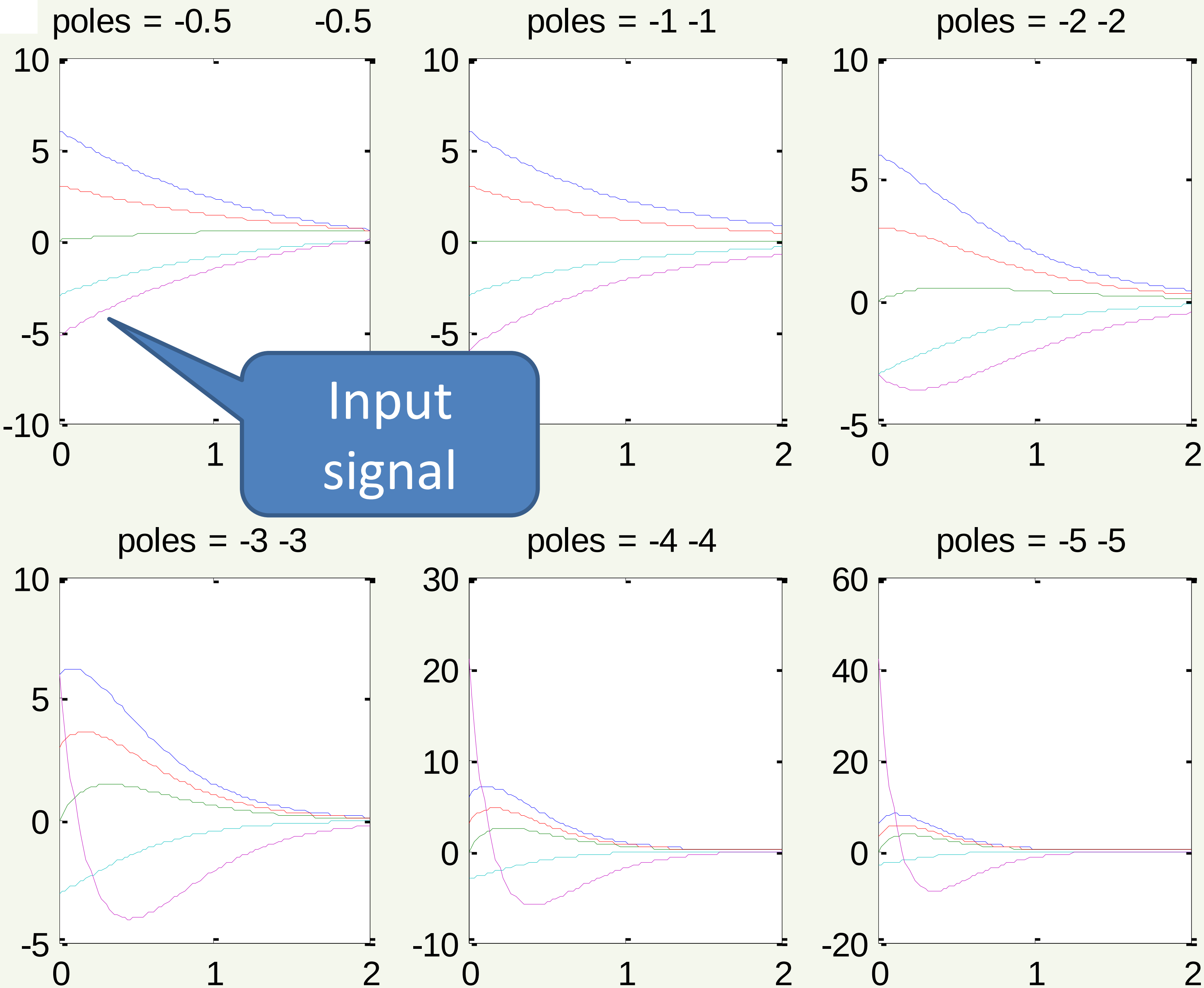
$$A=\begin{bmatrix}-1 & -2\\ 1 & 0\end{bmatrix};\quad B=\begin{bmatrix}1\\ 0\end{bmatrix};\quad C=\begin{bmatrix}1 & -1\\ 1 & 1\\ 1 & 0\\ 0 & 1\end{bmatrix};\quad K=\begin{bmatrix}k_1 & k_2\end{bmatrix}$$

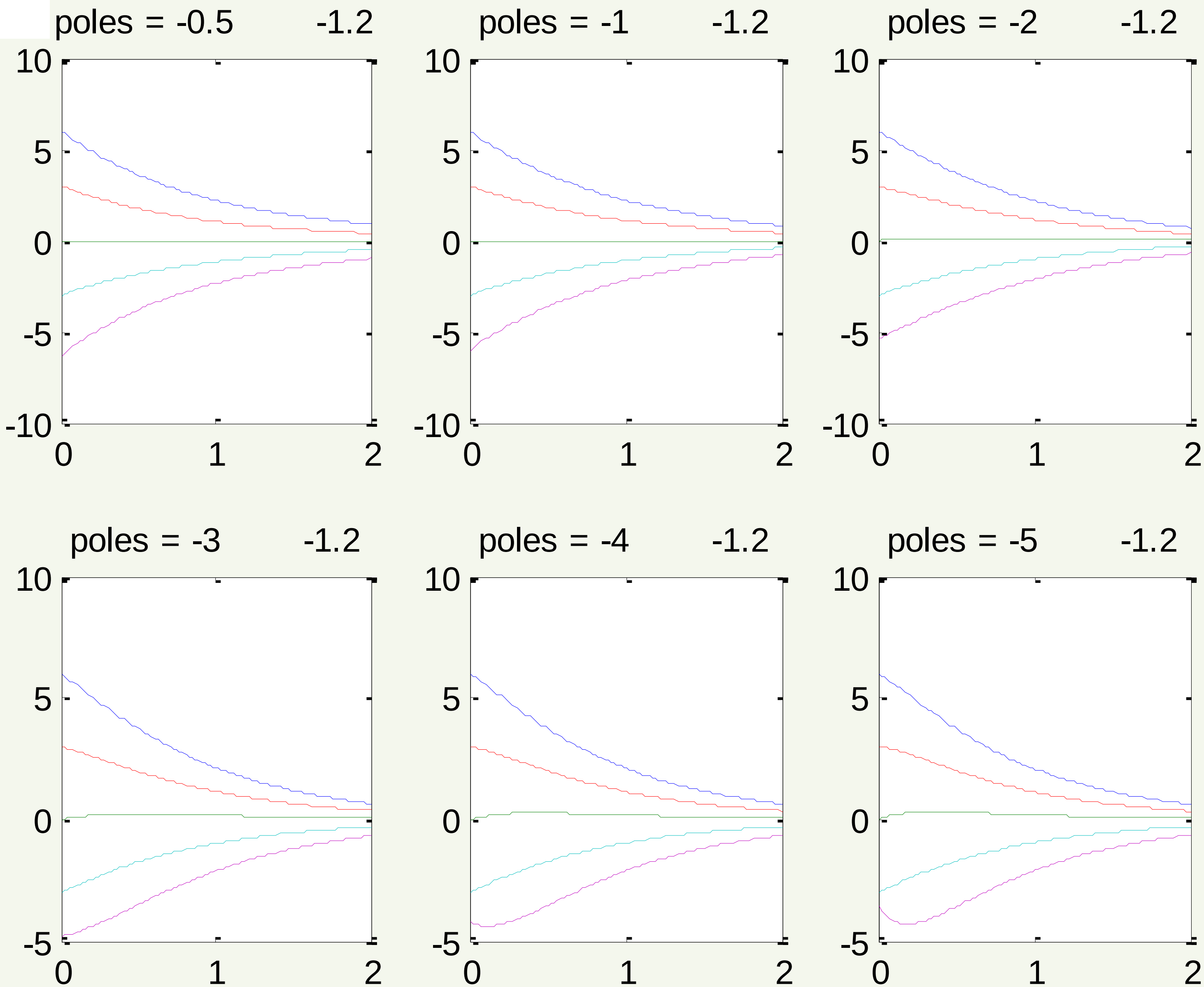
$$|\lambda I-(A-BK)| = \lambda^2+(1+k_1)\lambda+(2+k_2)=0$$

$$|\lambda I-(A-BK)| = (\lambda+p)(\lambda+q)$$

$$1+k_1=(p+q)$$

$$2+k_2=pq$$


poles = -0.5 -0.5
poles = -1 -1
poles = -2 -2
poles = -3 -3
poles = -4 -4
poles = -5 -5
10
5
0
-5
-10
30
20
-20
60
40
0
1
2
Input signal



Much less difference with one pole fixed at -1.2.

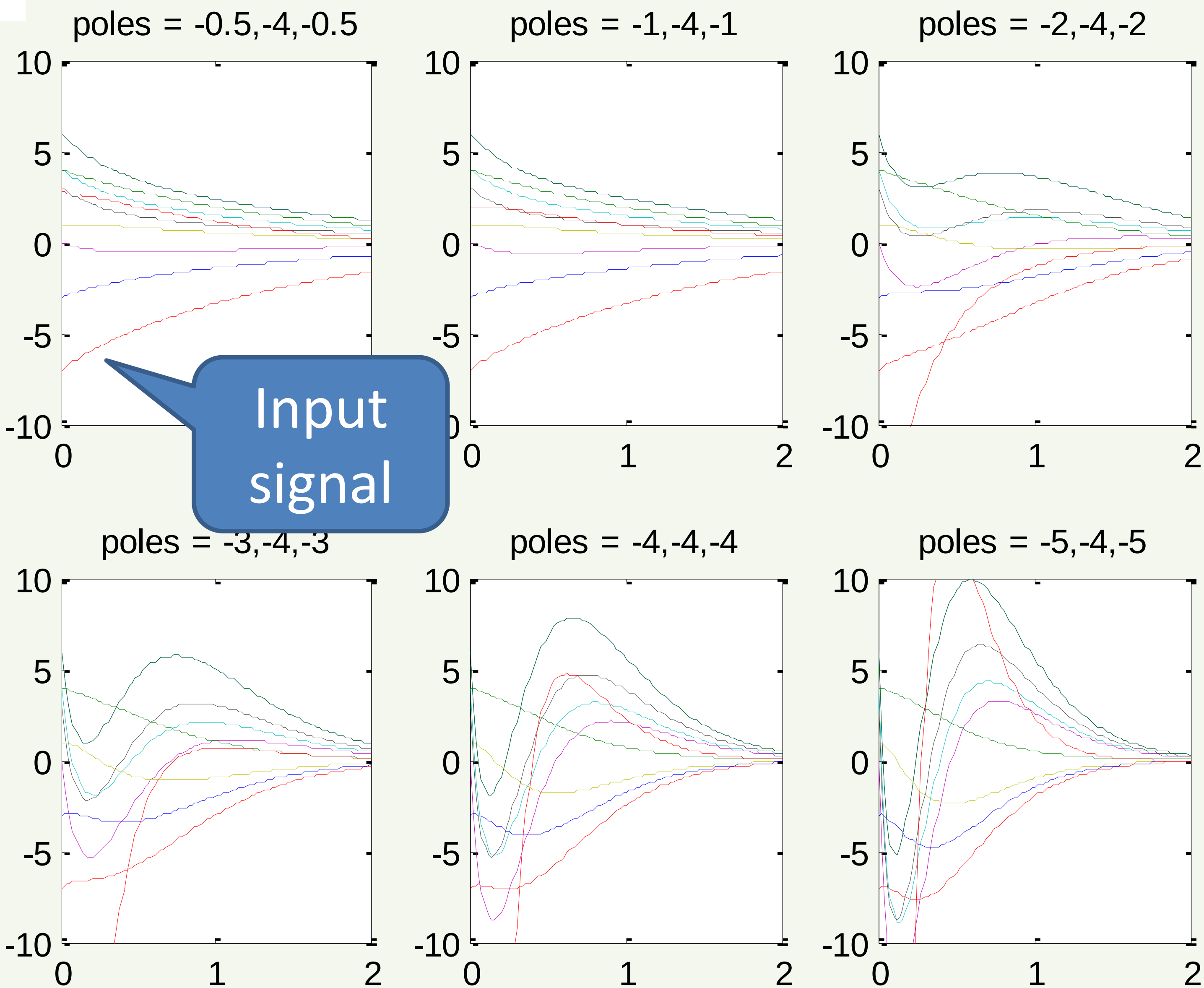
Example 2

$$A=\begin{bmatrix}-6 & -11 & -6\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix};\quad B=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad K=\begin{bmatrix}k_1 & k_2 & k_3\end{bmatrix}$$

$$|\lambda I-(A-BK)|=\lambda^3+(6+k_1)\lambda^2+(11+k_2)\lambda+(6+k_3)=0$$

$$|\lambda I-(A-BK)|=(\lambda+p)(\lambda+q)(\lambda+r)$$

$$6+k_1=p+q+r;\quad 11+k_2=pq+qr+pr;\quad 6+k_3=pqr$$



Behaviour hugely affected by targeted poles.

Pole selection

- With a small number of poles as in low order systems, one could use insights from root-loci or similar to suggest sensible closed-loop pole locations.
- However, with high order systems this becomes less obvious/systematic.
- A general piece of guidance is that poles should not be moved too far from the open-loop positions as this will probably necessitate aggressive inputs and also is likely to result in a sensitive feedback loop.

Summary $u = -Kx$

1. When a system is in controllable form, every coefficient of the closed-loop pole polynomial can be defined as desired using state feedback.
2. This means every closed-loop pole can be placed exactly as desired.
3. HOWEVER this does not imply knowledge of good places to put the poles. In general selecting fast poles may not imply good overall behaviour. A more systematic design approach is needed!
4. MOREOVER, we have not yet tackled tracking problems and ensuring the output reaches a specified target. Again, the required changes are not obvious.

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