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State-space feedback 8 dead beat control

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Introduction

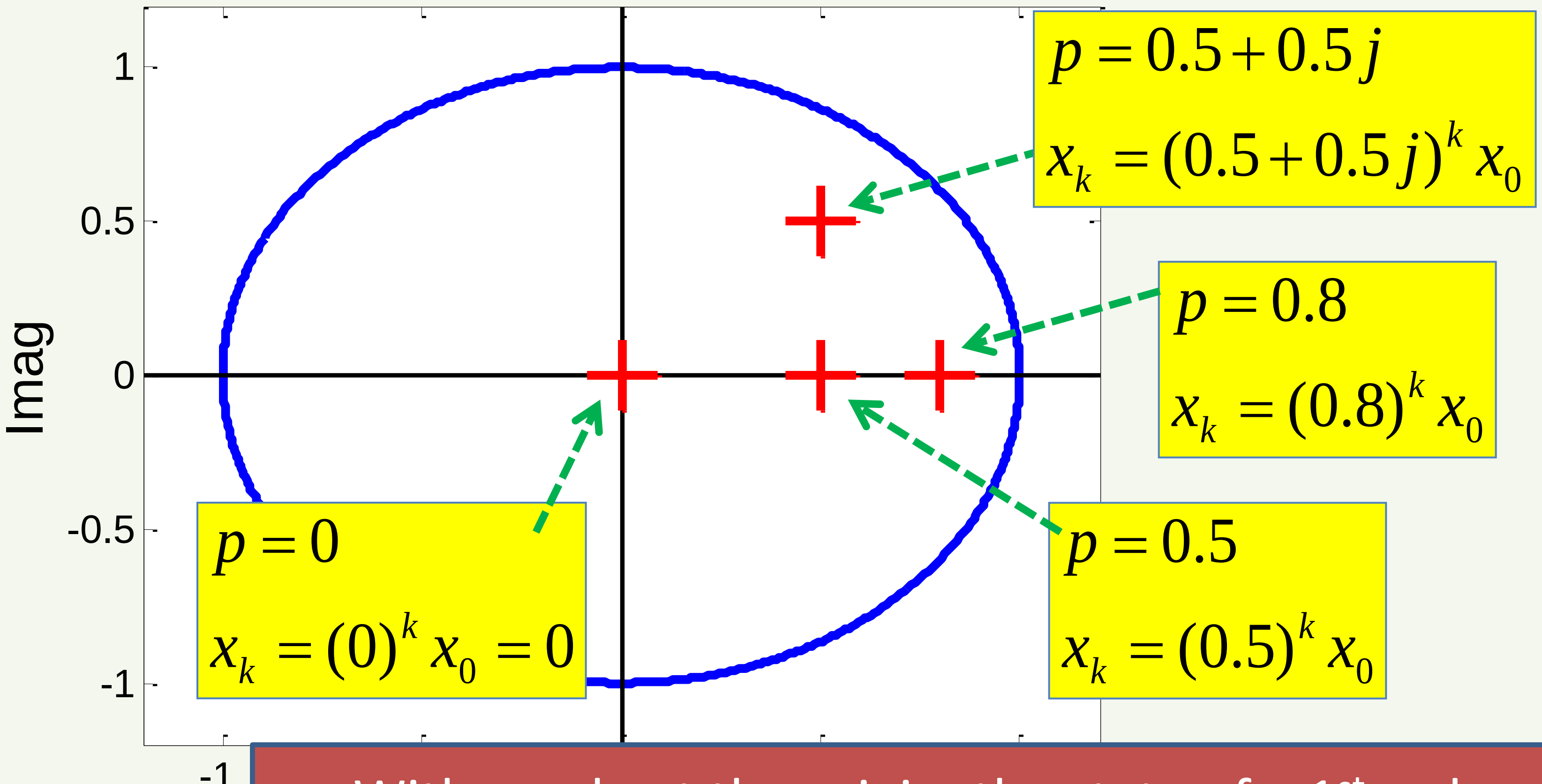
- The earlier videos introduced the concept of state feedback and demonstrated that it moves the poles.

$$\left\{ \begin{matrix} \dot{x} = Ax + Bu \\ u = -Kx \end{matrix} \right\} \quad \Rightarrow \quad \dot{x} = \underbrace{(A - BK)}_{\Phi} x$$

- It was shown that when a system is fully controllable, the poles can be placed arbitrarily, that is wherever the user desires.
- **This video looks specifically at the discrete case and asks, what happens if we choose to place all the poles on the origin?**

Discrete systems

Viewers are reminded of the links between pole positions and behaviour in the discrete case.



With a pole at the origin, the state of a 1st order process goes to zero in one sample!

Dead-beat control

Using pole placement, for a controllable system one can place all the poles at the origin.

This is called dead-beat control.

1. Is this a good idea?
2. What is the corresponding system behaviour?

First some examples will illustrate.

Examples

Illustrate the closed-loop state behaviour with dead-beat control.

$$A=\begin{bmatrix}-1 & -2\\ 1 & 0\end{bmatrix};\quad B=\begin{bmatrix}1\\ 0\end{bmatrix};\quad K=\begin{bmatrix}-1 & 2\end{bmatrix};\quad A-BK=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$

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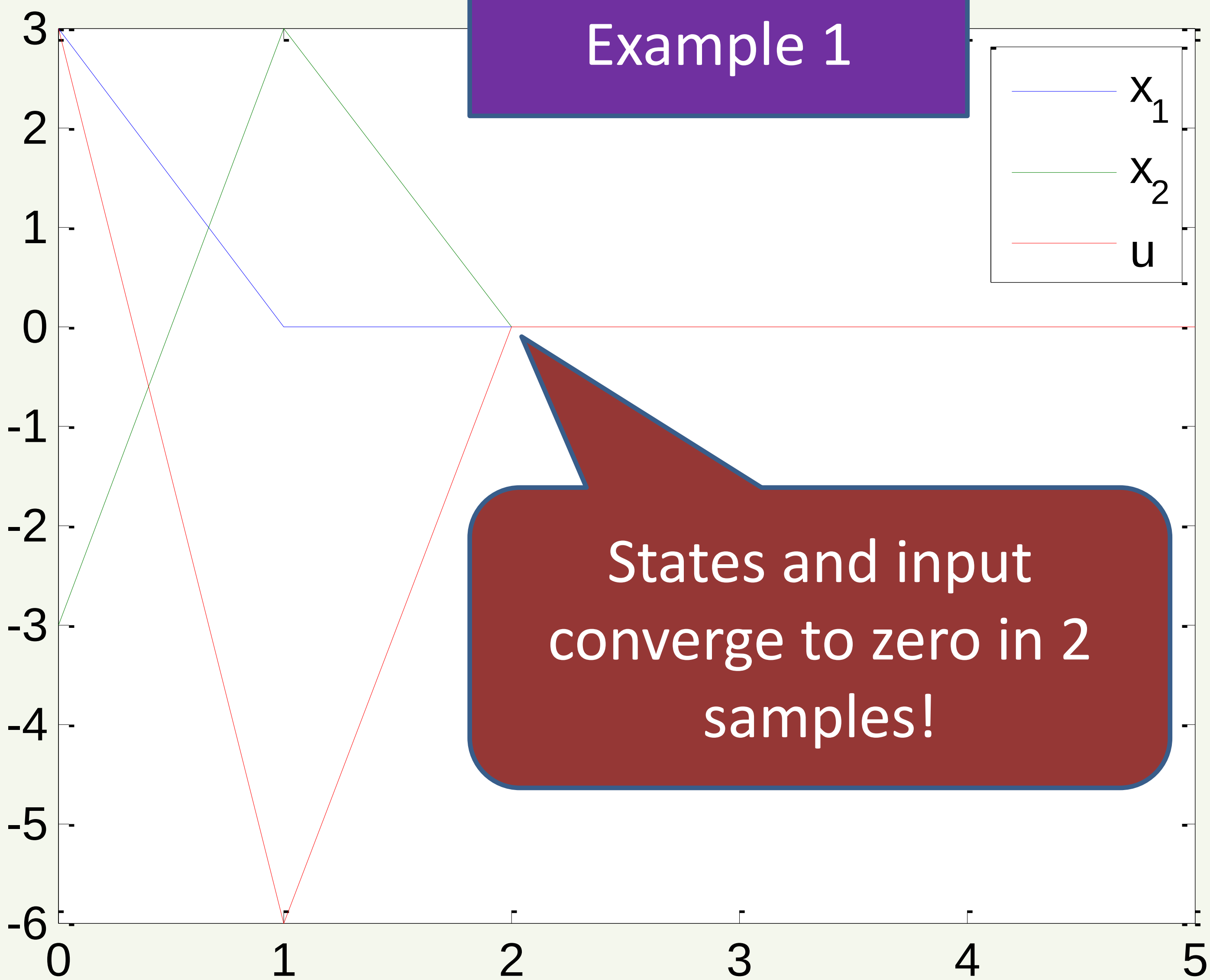
Example 1



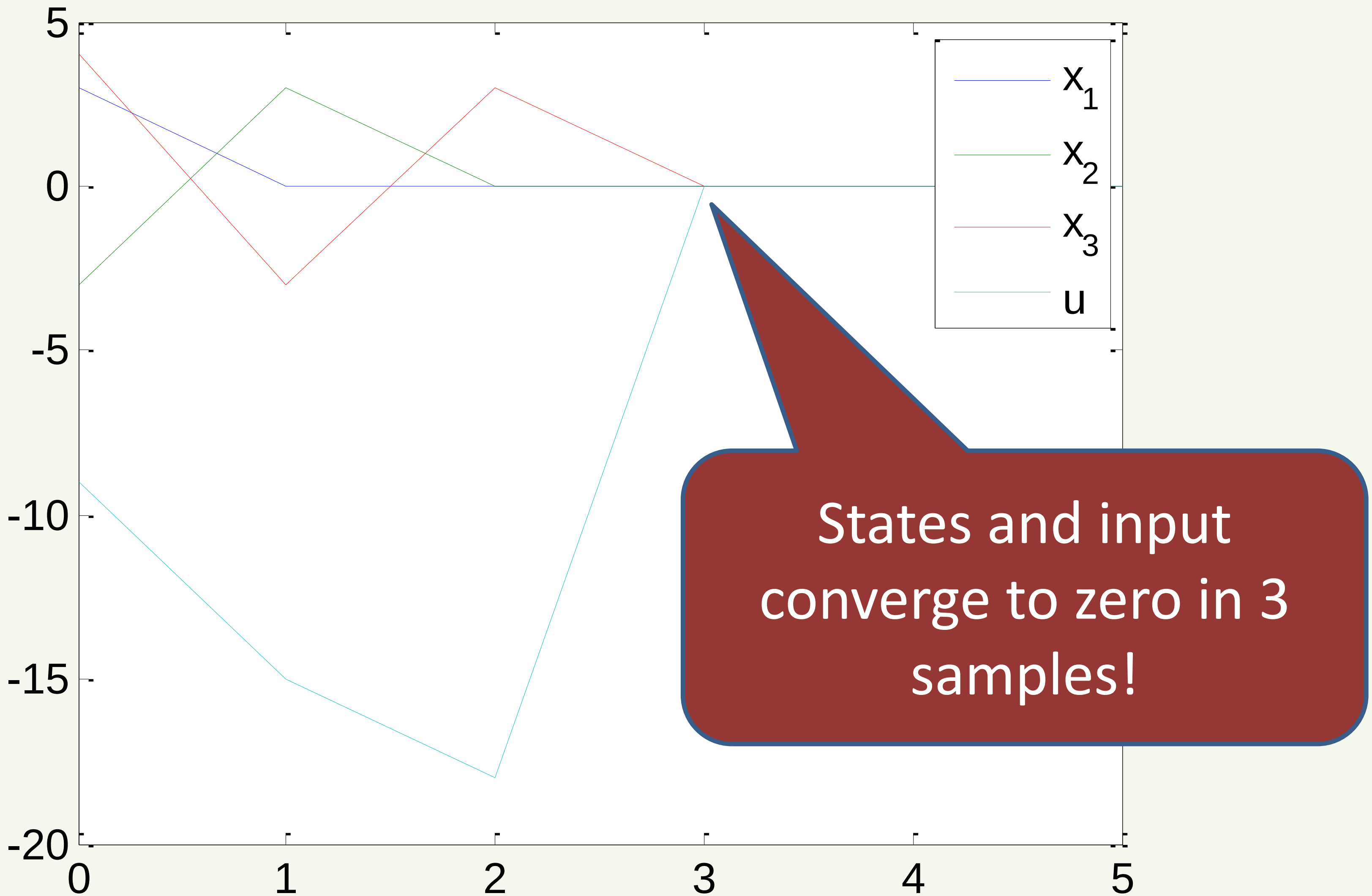
Examples

Illustrate the closed-loop state behaviour with dead-beat control.

$$A = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$K = \begin{bmatrix} -6 & -11 & -6 \end{bmatrix}; \quad A - BK = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Example 2


5
0
-5
-10
-15
-20
0
1
2
3
4
5
x1
x2
x3
u
States and input converge to zero in 3 samples!

Observations

With poles at the origin, the state trajectories seem to converge to zero in 'n' samples, where 'n' is the system order.

This is to be expected.

$$X(z) = (zI - A)^{-1}[Bu + x_o]$$

$$X_i(z) = \frac{n_i(z)}{d(z)}[\frac{Bu}{1-z^{-1}} + x_i(0)]$$

$$n_i(z) = b_1 z^{n-1} + b_2 z^{n-2} + \cdots + b_0$$

$$d(z) = z^n + a_1 z^{n-1} + \cdots + a_0$$

As all the poles are at the origin, then:

$$d(z) = z^n$$

$$\Rightarrow x_i(k) = [b_1, b_1 + b_2, \cdots, b_1 + \cdots + b_0, \cdots]u$$

EXAMPLE

$$A-BK=\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix};\quad (zI-A+BK)^{-1}=\begin{bmatrix}\frac{z^2}{z^3} & \cdots\\ \frac{z}{z^3} & \cdots\\ \frac{1}{z^3} & \cdots\end{bmatrix}$$

Interim summary

- Dead-beat control gives the fastest possible convergence, which is n-samples.
- Quicker is not possible due to the terms in the numerator polynomial.
- Dead-beat can be useful analysis tool as one is able to capture the entire dynamic in so few samples.

However, is such fast convergence a good thing?

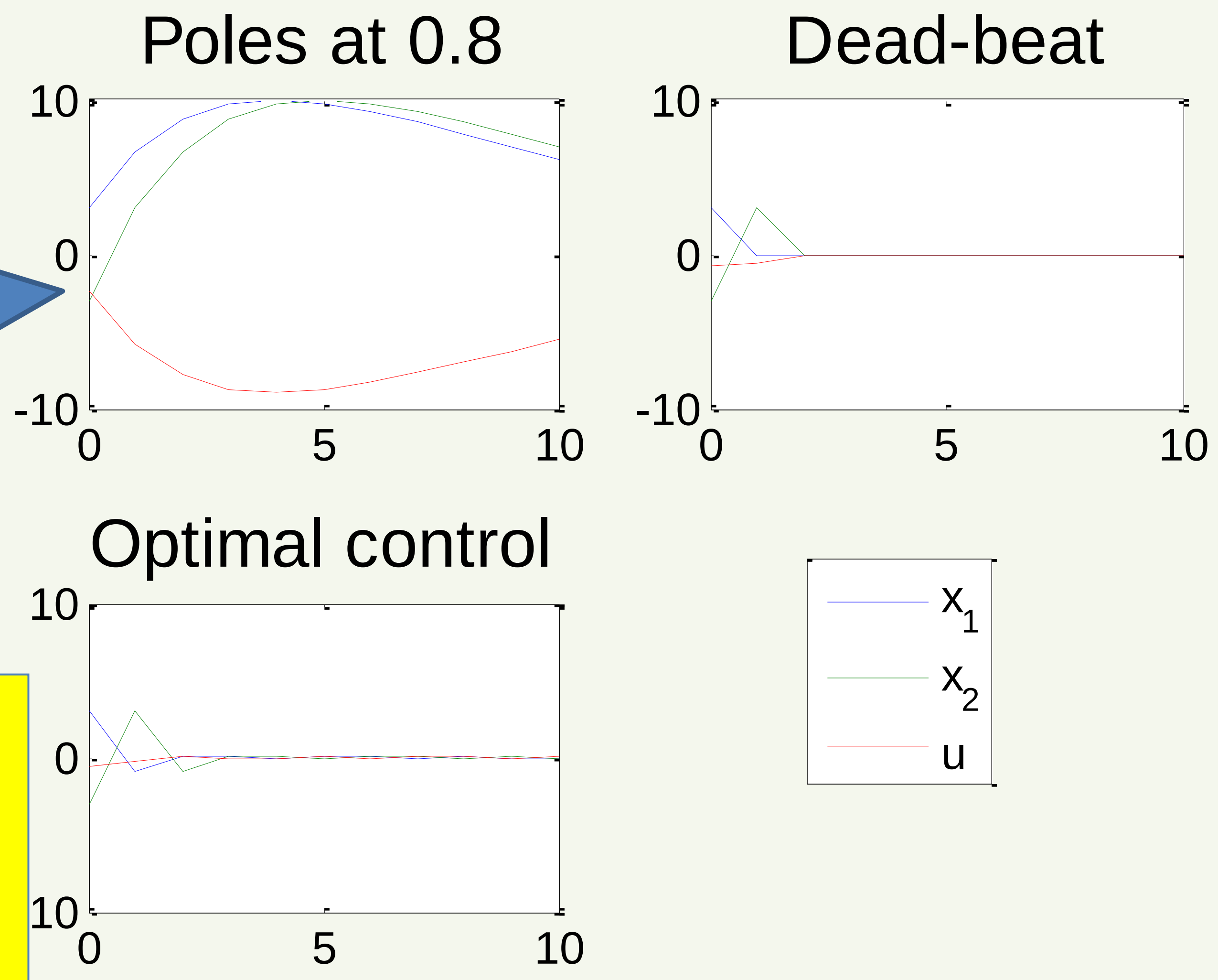
Performance

It is worth comparing the performance of dead-beat with optimal control and perhaps other pole placement choices.

We will not use specific criteria here but rather look at indicative characteristics.

Example 3

Pole placement relatively poor here.



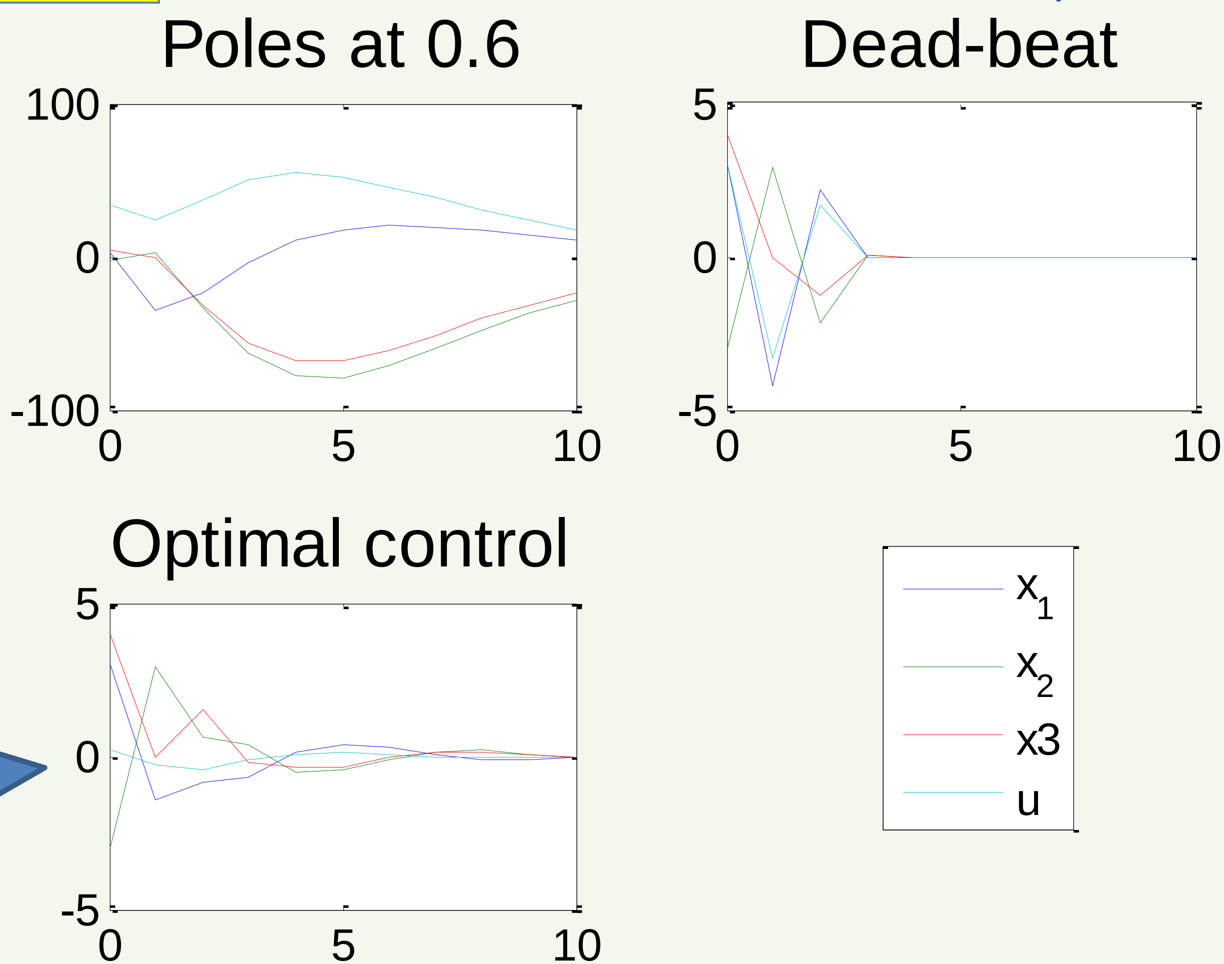
$$A = \begin{bmatrix} -1.8 & -0.82 \\ 1 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A=\begin{bmatrix}0 & -0.4 & -0.6\\ 1 & 0.7 & 0.5\\ 1 & 1 & 0\end{bmatrix}$$

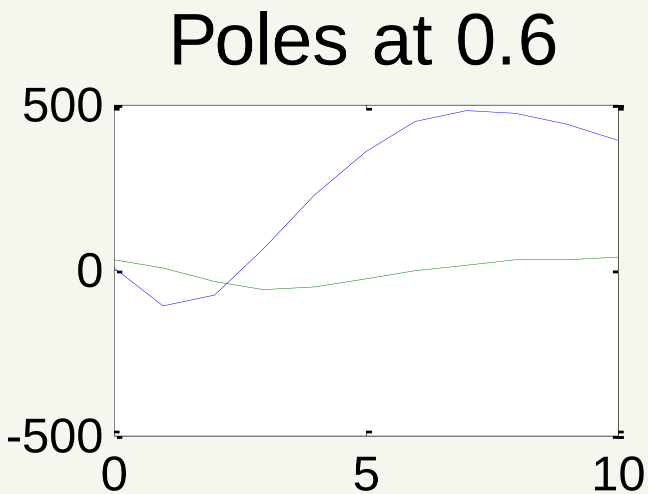
Example 4

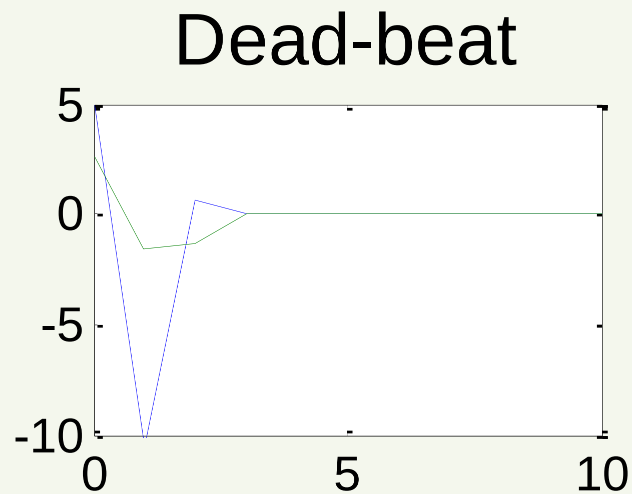
Aggressive input

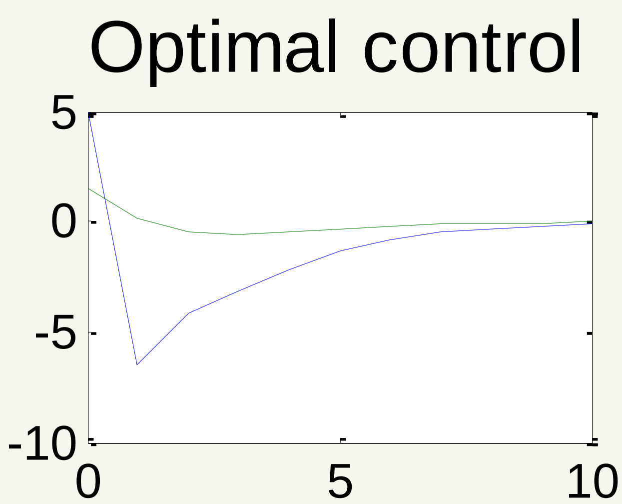


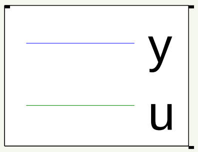
Relatively low input activity

Example 5 – 4^{th} order









Dead-beat much more aggressive with the input.

Summary $u = -Kx$

1. Dead-beat control is defined for discrete systems as a pole placement design placing all the poles at the origin.
2. Responses converge in 'n' samples, n being the system order.
3. Can be a useful benchmark and analysis tool, but rarely gives satisfactory performance.
4. Implicitly poorly conditioned, sensitive and likely to demand over active inputs.

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