



State-space feedback & dead beat control

J A Rossiter

Introduction

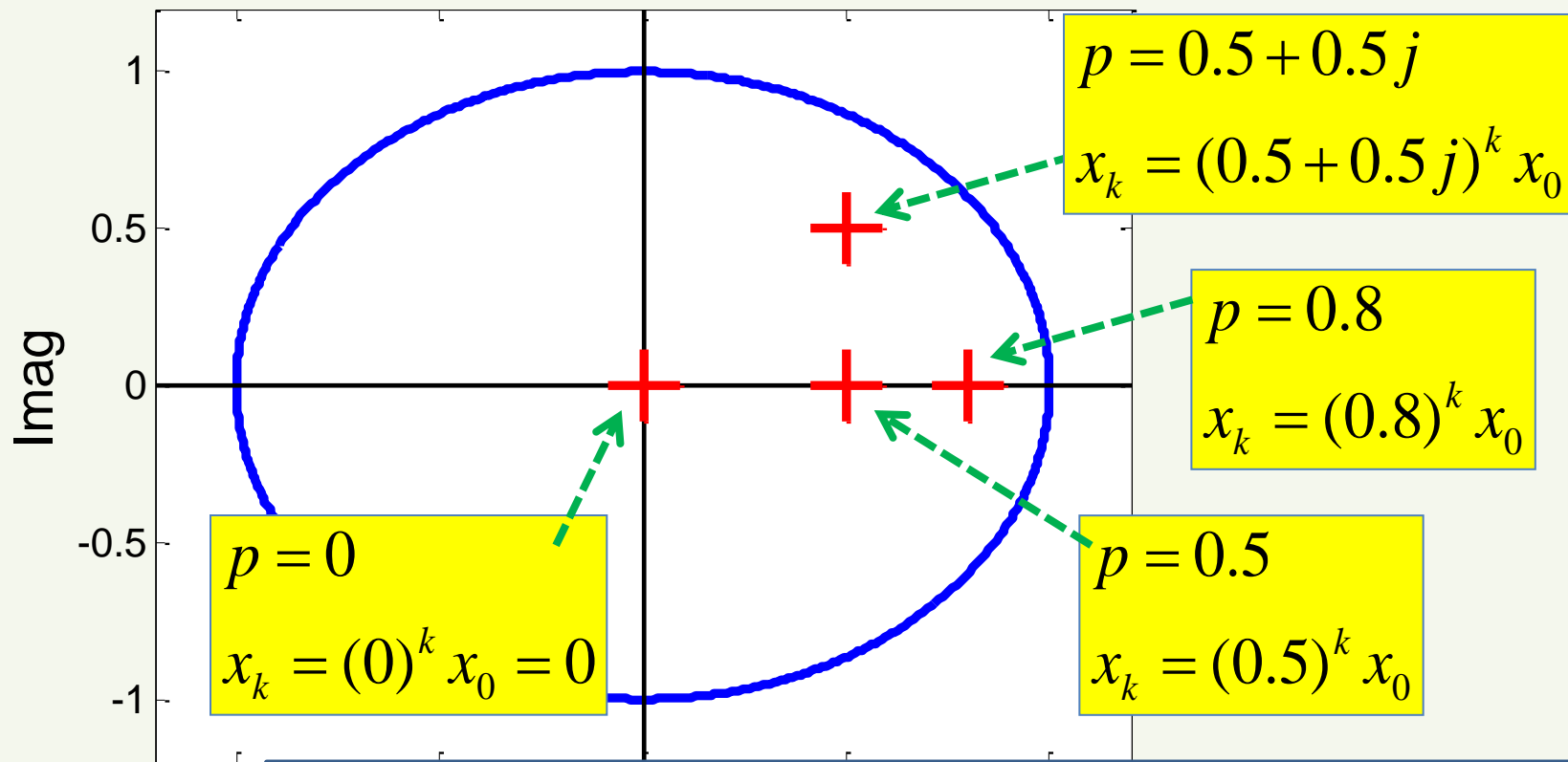
- The earlier videos introduced the concept of state feedback and demonstrated that it moves the poles.

$$\left\{ \begin{array}{l} \dot{x} = Ax + Bu \\ u = -Kx \end{array} \right\} \Rightarrow \dot{x} = \underbrace{(A - BK)}_{\Phi} x$$

- It was shown that when a system is fully controllable, the poles can be placed arbitrarily, that is wherever the user desires.
- **This video looks specifically at the discrete case and asks, what happens if we choose to place all the poles on the origin?**

Discrete systems

Viewers are reminded of the links between pole positions and behaviour in the discrete case.



With a pole at the origin, the state of a 1st order process goes to zero in one sample!

Dead-beat control

Using pole placement, for a controllable system one can place all the poles at the origin.

This is called dead-beat control.

1. Is this a good idea?
2. What is the corresponding system behaviour?

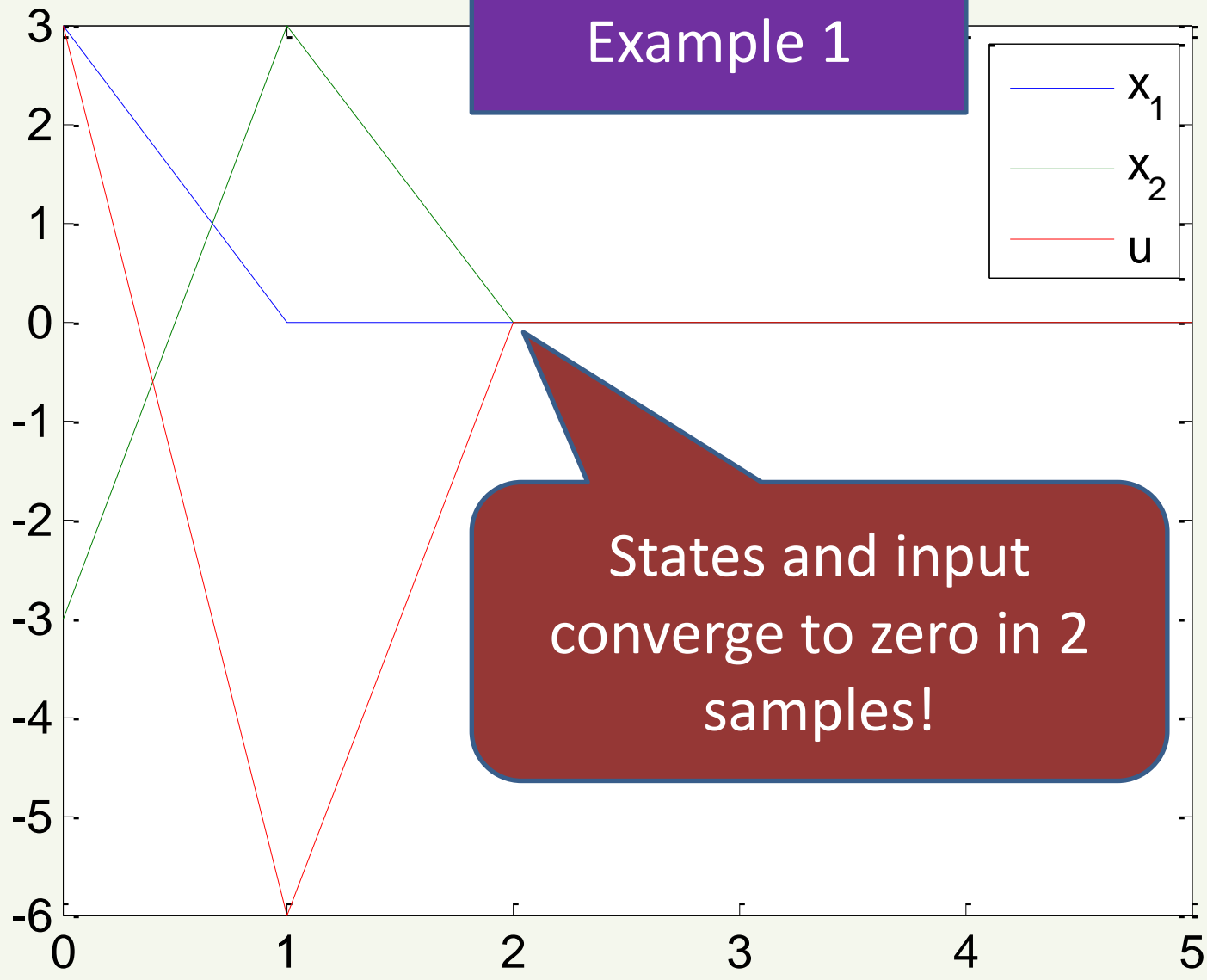
First some examples will illustrate.

Examples

Illustrate the closed-loop state behaviour with dead-beat control.

$$A = \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad K = [-1 \quad 2]; \quad A - BK = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Example 1



States and input converge to zero in 2 samples!

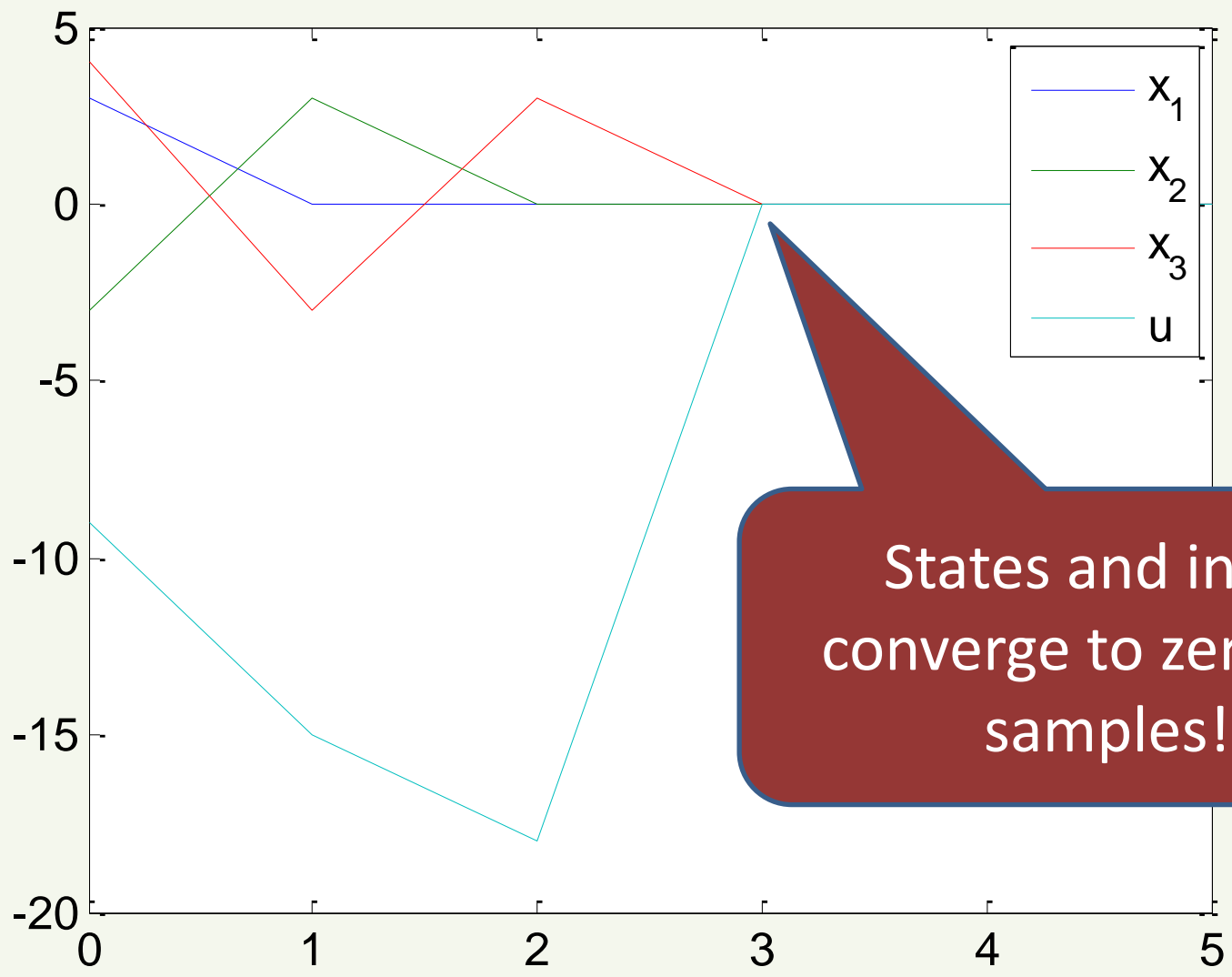
Examples

Illustrate the closed-loop state behaviour with dead-beat control.

$$A = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$K = [-6 \quad -11 \quad -6]; \quad A - BK = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Example 2



States and input converge to zero in 3 samples!

Observations

With poles at the origin, the state trajectories seem to converge to zero in 'n' samples, where 'n' is the system order.

This is to be expected.

$$X(z) = (zI - A)^{-1}[Bu + x_o]$$

$$X_i(z) = \frac{n_i(z)}{d(z)} \left[\frac{Bu}{1 - z^{-1}} + x_i(0) \right]$$

$$n_i(z) = b_1 z^{n-1} + b_2 z^{n-2} + \dots + b_0$$

$$d(z) = z^n + a_1 z^{n-1} + \dots + a_0$$

As all the poles are at the origin, then:

$$d(z) = z^n$$

$$\Rightarrow x_i(k) = [b_1, b_1 + b_2, \dots, b_1 + \dots + b_0, \dots]u$$

EXAMPLE

$$A - BK = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad (zI - A + BK)^{-1} = \begin{bmatrix} \frac{z^2}{z^3} & \dots \\ \frac{z}{z^3} & \dots \\ 1 & \dots \\ \frac{1}{z^3} & \dots \end{bmatrix}$$

Interim summary

- Dead-beat control gives the fastest possible convergence, which is n -samples.
- Quicker is not possible due to the terms in the numerator polynomial.
- Dead-beat can be useful analysis tool as one is able to capture the entire dynamic in so few samples.

However, is such fast convergence a good thing?

Performance

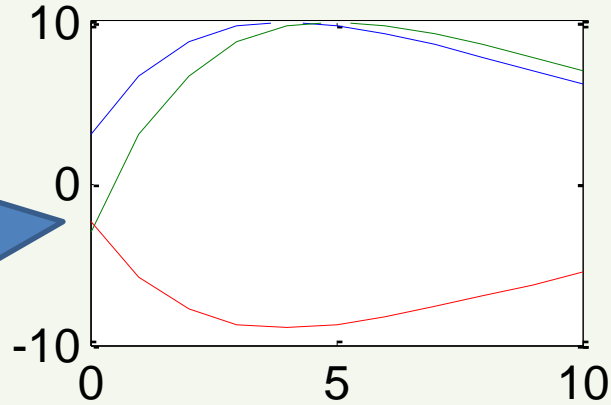
It is worth comparing the performance of dead-beat with optimal control and perhaps other pole placement choices.

We will not use specific criteria here but rather look at indicative characteristics.

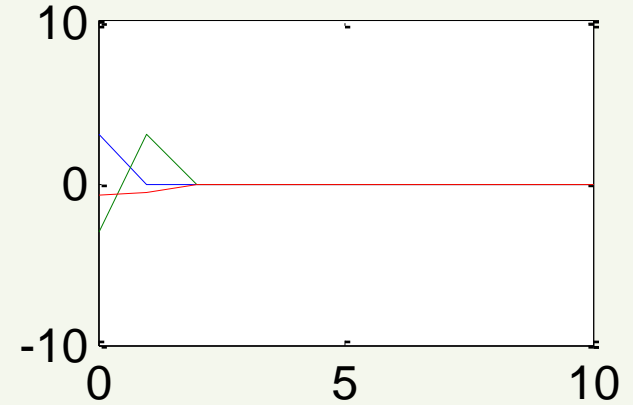
Example 3

Pole placement relatively poor here.

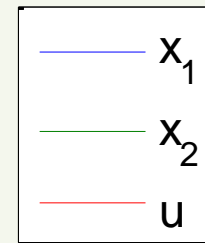
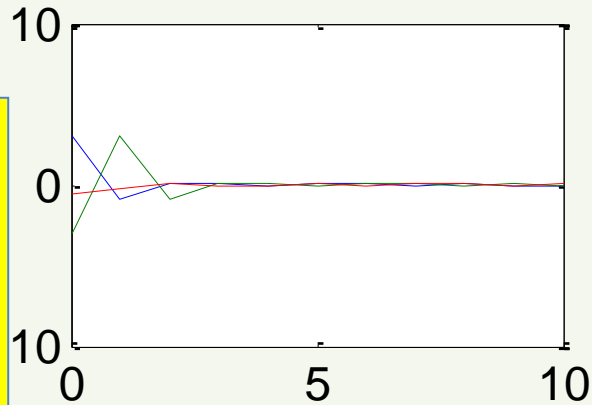
Poles at 0.8



Dead-beat



Optimal control



$$A = \begin{bmatrix} -1.8 & -0.82 \\ 1 & 0 \end{bmatrix};$$

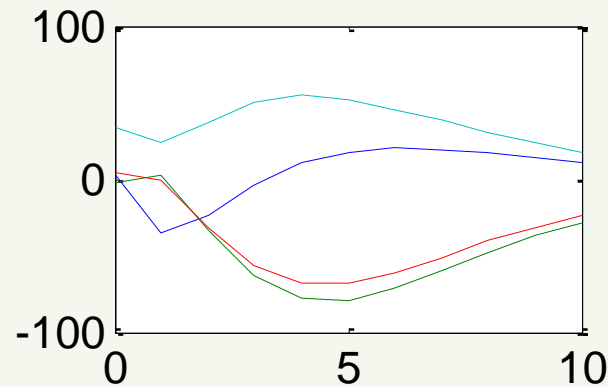
$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -0.4 & -0.6 \\ 1 & 0.7 & 0.5 \\ 1 & 1 & 0 \end{bmatrix}$$

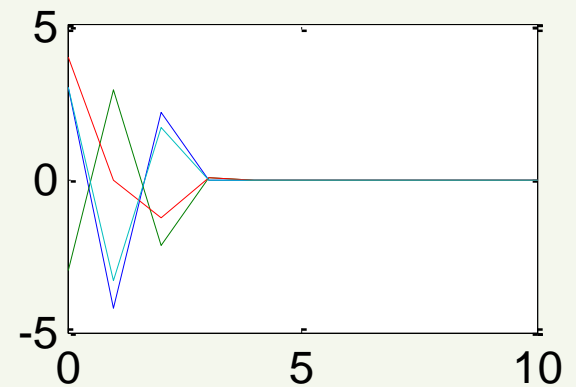
Example 4

Aggressive
input

Poles at 0.6

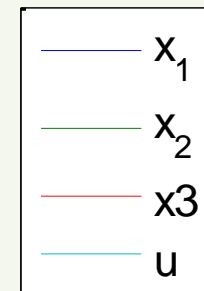
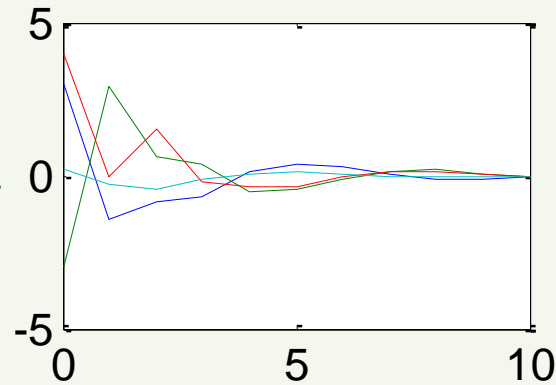


Dead-beat



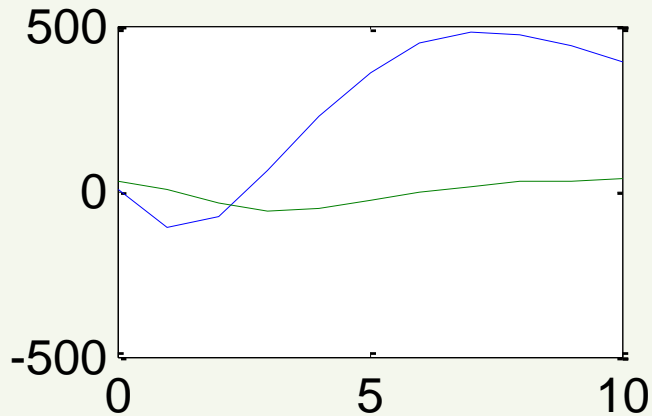
Relatively
low input
activity

Optimal control

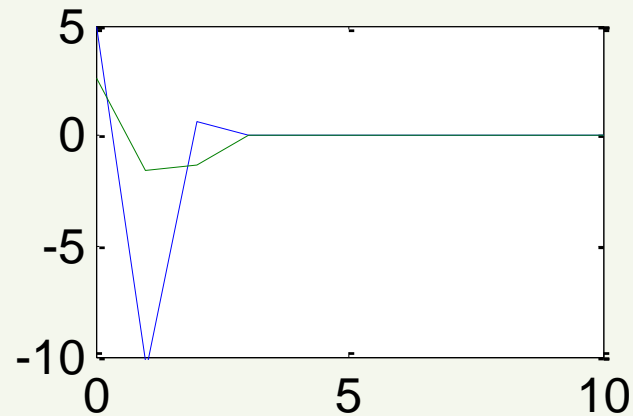


Example 5 – 4th order

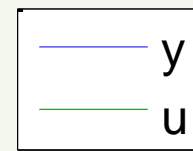
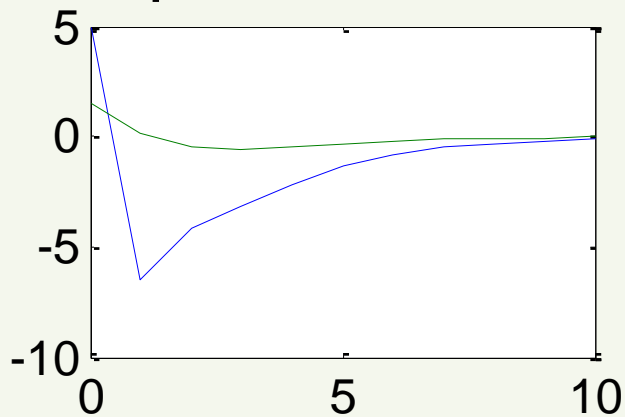
Poles at 0.6



Dead-beat



Optimal control



Dead-beat much more aggressive with the input.

Summary

$$u = -Kx$$

1. Dead-beat control is defined for discrete systems as a pole placement design placing all the poles at the origin.
2. Responses converge in 'n' samples, n being the system order.
3. Can be a useful benchmark and analysis tool, but rarely gives satisfactory performance.
4. Implicitly poorly conditioned, sensitive and likely to demand over active inputs.



Anthony Rossiter
Department of Automatic Control and
Systems Engineering
University of Sheffield
www.shef.ac.uk/acse

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