State-space observers 2
basic structure

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Introduction

The previous video introduced the concept of an observer.

1. An observer combines different forms of knowledge, facts and measurements to make inferences about a system state.

2. Typically an observer combines:
   a) Knowledge of past inputs.
   b) Available measurements of outputs.
   c) Knowledge of model parameters and thus system dynamics.

3. Next, we propose a mechanism for using this information systematically.
The underlying system has known model and behaviour.

Given knowledge of $u(t)$ and an initial condition, we can predict the evolution of $x(t)$ [and $y(t)$].

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &=Cx
\end{align*}
\]

\[
x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} Bu \, d\tau
\]
A comparison of predicted model outputs and the real system outputs must be linked to initialisation errors.

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

\[
x(t) = e^{At}x(0) + \int_{0}^{t} e^{A(t-\tau)} Bu d\tau
\]

In the absence of parameter errors, any errors in our knowledge of \(x(t)\) relate solely to initialisation errors.
Interim summary

Simulate a model (state z) in parallel with the process.

\[
y(t) = Ce^{At}x(0) + C \int_{0}^{t} e^{A(t-\tau)} Bud \tau
\]

\[
y_m(t) = Ce^{At}z(0) + C \int_{0}^{t} e^{A(t-\tau)} Bud \tau
\]

Assuming stable dynamics, the difference 
\[y - y_m \xrightarrow{\text{as } t \to \infty} 0\]
Interim conclusion

If the model parameters and input $u(t)$ are known exactly, then:

\[ t > t_1 \implies e^{At} \approx 0 \]

\[ t > t_1 \implies \{ y(t) \approx y_m(t), x(t) \approx z(t) \} \]

A simple observer is an open-loop model simulation in parallel with the actual process.
Caviats

In practice these assumptions fail.

1. The model parameters are not known exactly.
2. There are also exogenous signals affecting a real system such as disturbances.
3. Assumes stable open-loop dynamics.

Therefore an open-loop simulation is inadequate because it makes no use of system information and system measurements to re-calibrate.

We need to make use of measurement information.
Parallel simulation

Simulate a model (state $z$) in parallel with the process.

$$y(t) = C e^{At} x(0) + C \int_{0}^{t} e^{A(t-\tau)} B u d\tau + \hat{C} \int_{0}^{t} e^{A(t-\tau)} \hat{B} v d\tau$$

$$y_m(t) = C_m e^{A_m t} z(0) + C_m \int_{0}^{t} e^{A_m (t-\tau)} B_m u d\tau$$

Even with stable dynamics, the difference $y - y_m \to N \neq 0$, as $t \to \infty$ due to parameter errors and unknown signal $v(t)$.
Requirement

The parallel model simulation needs to be re-calibrated to ensure it converges to match the true model states, irrespective of \( v(t) \).

A logical mechanism for doing this is use the error between the model and system outputs and force this error to zero.

\[
\begin{align*}
e(t) &= y(t) - y_m(t); \quad \lim_{t \to \infty} e(t) = 0 \\
e_x(t) &= x(t) - z(t); \quad \lim_{t \to \infty} e_x(t) = 0
\end{align*}
\]

What mechanism will embed this requirement systematically?
Proposal

Form a model for which the state error is the state of the system and then ensure this model has stable dynamics so that its state converges to zero.

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
\dot{z} &= Az + Bu + L(y - y_m) \\
\end{align*}
\]

\[
\begin{align*}
y &= Cx \\
y_m &= Cz \\
\end{align*}
\]

\[
\begin{align*}
\dot{e}_x &= \dot{x} - \dot{z} = A(x - z) + Bu - Bu + L(Cx - Cz) \\
y - y_m &= Ce_x \\
\end{align*}
\]

\[
\dot{e}_x = (A - LC)e_x
\]
Remark

Viewers will note that the proposed system is derived by simulating both the process and model and parallel but also,

Augmenting the model dynamics with a term based on the output error between the two systems.

\[
\begin{align*}
\dot{z} &= Az + Bu + L(y - y_m) \\
2 \quad y_m &= Cz,
\end{align*}
\]

\[ \dot{e}_x = (A - LC)e_x \]

Stability of A-LC ensures the errors converge.
Conclusion

Derived the based structure of a state observer.

1. Simulate a process model in parallel, but modify the dynamics with an error term linked to the measured system and model outputs.

\[
\begin{aligned}
\dot{z} &= Az + Bu + L(y - y_m) \\
y_m &= Cz \\
\end{aligned}
\]

2. It is straightforward to show that the error state is governed by.

\[
\dot{x} - \dot{z} = (A - LC)(x - z)
\]
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