State-space observers 2 basic structure

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Introduction

The previous video introduced the concept of an observer.

1. An observer combines different forms of knowledge, facts and measurements to make inferences about a system state.
2. Typically an observer combines:
   a) Knowledge of past inputs.
   b) Available measurements of outputs.
   c) Knowledge of model parameters and thus system dynamics.
3. Next, we propose a mechanism for using this information systematically.

Context

The underlying system has known model and behaviour.

Given knowledge of u(t) and an initial condition, we can predict the evolution of x(t) [and y(t)].

$$\left\{\begin{array}{c} \dot{x} = Ax + Bu \\ y = Cx \end{array}\right\}$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bud\tau$$

Assume A,B,C, known exactly

Known exactly

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Context

A comparison of predicted model outputs and the real system outputs must be linked to initialisation errors.

$$\left\{ \begin{matrix} \dot{x} = Ax + Bu \\ y = Cx \end{matrix} \right\}$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} Bud\tau$$

x(0) not known

In the absence of parameter errors, any errors in our knowledge of x(t) relate solely to initialisation errors.

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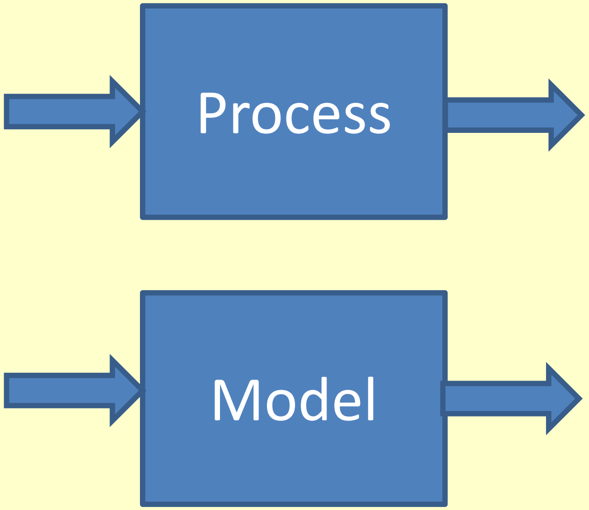
Interim summary

Simulate a model (state z) in parallel with the process.



$$y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bud\tau$$

$$y_m(t) = Ce^{At}z(0) + C\int_0^t e^{A(t-\tau)}Bud\tau$$

Assuming stable dynamics, the difference $y-y_m \rightarrow 0$, as $t \rightarrow \infty$

Interim conclusion

If the model parameters and input u(t) are known exactly, then:

$$t > t_1 \quad \Rightarrow \quad e^{At} \approx 0$$

$$t > t_1 \quad \Rightarrow \quad \{y(t) \approx y_m(t), x(t) \approx z(t)\}$$

A simple observer is an open-loop model simulation in parallel with the actual process.

Caviats

In practice these assumptions fail.

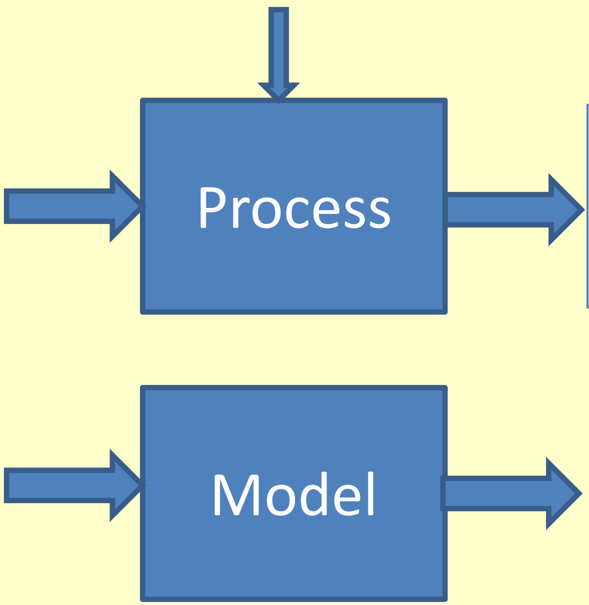
1. The model parameters are not known exactly.
2. There are also exogenous signals affecting a real system such as disturbances.
3. Assumes stable open-loop dynamics.

Therefore an open-loop simulation is inadequate because it makes no use of system information and system measurements to re-calibrate.

We need to make use of measurement information.

Parallel simulation

Simulate a model (state z) in parallel with the process.



Disturbance effects

$$y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bud\tau + \hat{C}\int_0^t e^{A(t-\tau)}\hat{B}vd\tau$$

$$y_m(t) = C_m e^{A_m t}z(0) + C_m\int_0^t e^{A_m(t-\tau)}B_m ud\tau$$

Even with stable dynamics, the difference $y-y_m \rightarrow N \neq 0$, as $t \rightarrow \infty$ due to parameter errors and unknown signal v(t).

Requirement

The parallel model simulation needs to be re-calibrated to ensure it converges to match the true model states, irrespective of v(t).

A logical mechanism for doing this is use the error between the model and system outputs and force this error to zero.

$$e(t) = y(t) - y_m(t); \quad \lim_{t\to\infty} e(t) = 0$$

$$e_x(t) = x(t) - z(t); \quad \lim_{t\to\infty} e_x(t) = 0$$

What mechanism will embed this requirement systematically?

Proposal

Form a model for which the state error is the state of the system and then ensure this model has stable dynamics so that its state converges to zero.

$$\begin{Bmatrix} \dot{x} = Ax + Bu \\ y = Cx \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{z} = Az + Bu + L(y - y_m) \\ y_m = Cz \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{e}_x = \dot{x} - \dot{z} = A(x - z) + Bu - Bu + L(Cx - Cz) \\ y - y_m = Ce_x \end{Bmatrix}$$

$$\dot{e}_x = (A - LC)e_x$$

Remark

Viewers will note that the proposed system is derived by simulating both the process and model and parallel but also,

Augmenting the model dynamics with a term based on the output error between the two systems.

$$\left\{ \begin{array}{c} \dot{z} = Az + Bu + L(y - y_m) \\ y_m = Cz \end{array} \right\}$$

$$\dot{e}_x = (A - LC)e_x$$

Stability of A-LC ensures the errors converge.

Conclusion

Derived the based structure of a state observer.

1. Simulate a process model in parallel, but modify the dynamics with an error term linked to the measured system and model outputs.

$$\left\{ \begin{array}{c} \dot{z} = Az + Bu + L(y - y_m) \\ y_m = Cz \end{array} \right\}$$

2. It is straightforward to show that the error state is governed by.

$$\dot{x} - \dot{z} = (A - LC)(x - z)$$

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