The University Of Sheffield.

State-space observers 4 system stability

J A Rossiter

Introduction

The previous video demonstrated that one could derive an observer gain **L** to give the desired state estimation error dynamics.

$$\left\{ \begin{array}{c} \dot{x} = Ax + Bu \\ y = Cx \end{array} \right\}$$

$$\left\{ \begin{array}{c} \dot{z} = Az + Bu + L(y - y_m) \\ y_m = Cz \end{array} \right\}$$

$$\dot{x} - \dot{z} = (A - LC)(x - z)$$

However, in practice an observer gives state estimates used by state feedback for system control.

What is the impact on overall stability?

Overall structure

Combining the observer, state feedback and system dynamics requires a number of parallel models.

$$\left\{\begin{matrix} \dot{x} = Ax + Bu \\ y = Cx \end{matrix}\right\}$$

$$\left\{\begin{matrix} \dot{z} = Az + Bu + L(y - y_m) \\ y_m = Cz \end{matrix}\right\}$$

$$u = -Kz$$

Is the overall system of equations stable?

For now, assume no parameter uncertainty.

Combine equations

The easiest way to analyse the coupled dynamic equations is to construct a single equivalent state space model with states x and z.

$$\begin{cases} \dot{x} = Ax + Bu; \quad u = -Kz, \quad y = Cx \\ \dot{z} = Az + Bu + L(y - y_m); \quad y_m = Cz \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}$$

Next, use a similarity transform to help expose the underlying modes.

Similarity transformation

One can easily expose the underlying modes.

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}$$

$$T = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}; \quad T^{-1} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix};$$

$$T \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} T^{-1} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix}$$

Transformed matrix is now block diagonal

Overall modes

Key modes are taken from the eigenvalues of the transformation matrix.

$$A_c = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}$$

$$|\lambda I - A_c| = 0 \quad \Rightarrow \quad \begin{Bmatrix} |\lambda I - A + BK| = 0 \\ |\lambda I - A + LC| = 0 \end{Bmatrix}$$

Closed-loop poles when combining state feedback with an observer are the poles from the state feedback design combined with the poles from the observer design.

S

The University Of Sheffield.

Remark – separation principle

One can design the state feedback and observers independently one of an other without any impact on overall closed-loop stability.

NOTE however:

- We have assumed no parameter uncertainty.
- We have not considered the impact of the combined design on overall behaviour.

MATLAB ILLUSTRATIONS

We will compare performance with an observer and assuming states are measureable.

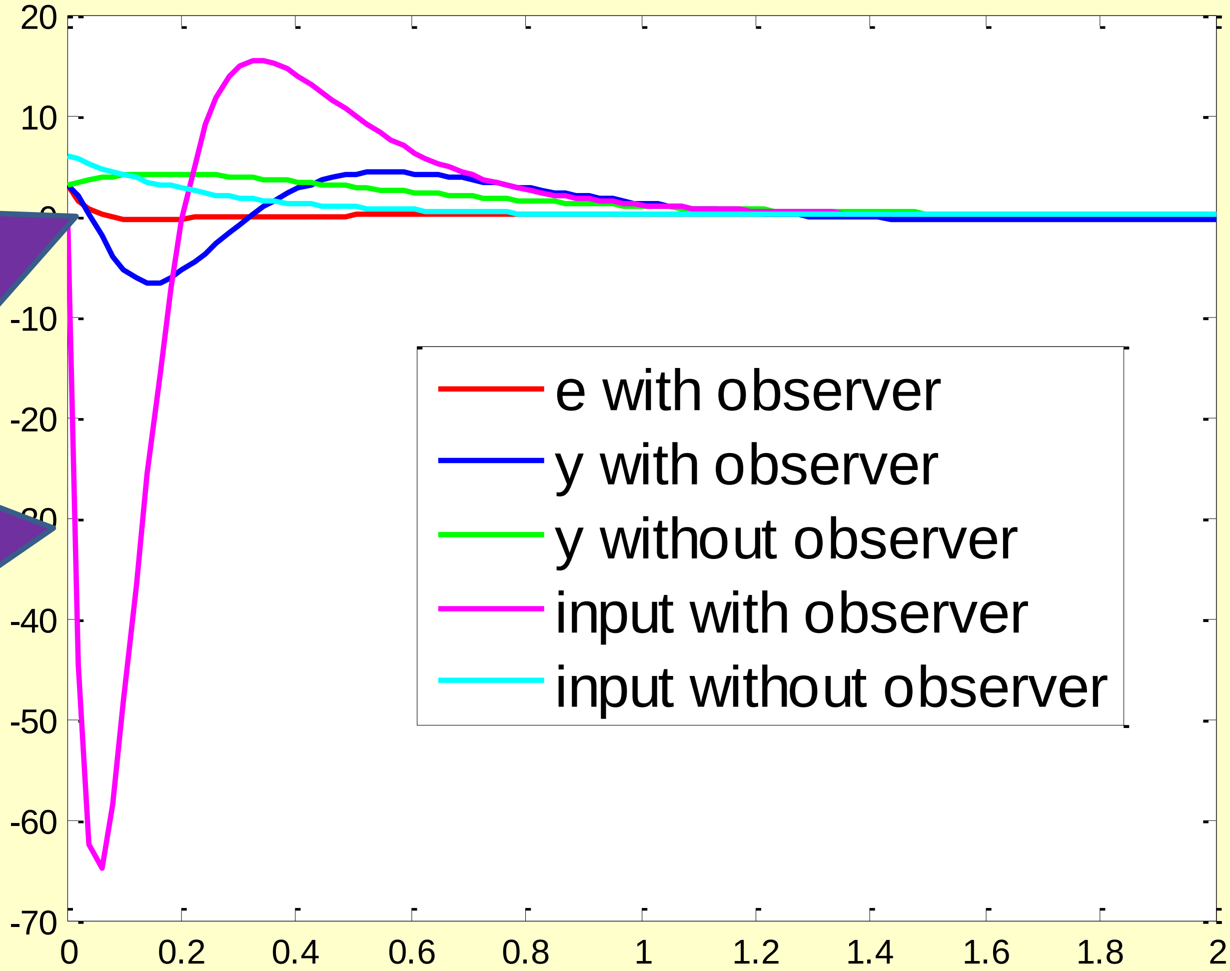
How does overall behaviour change with different choices of observer poles?

We will not look at the impact of parameter uncertainty.

Make observer poles 10x faster than system poles so that state errors converge quickly

Fast error convergence

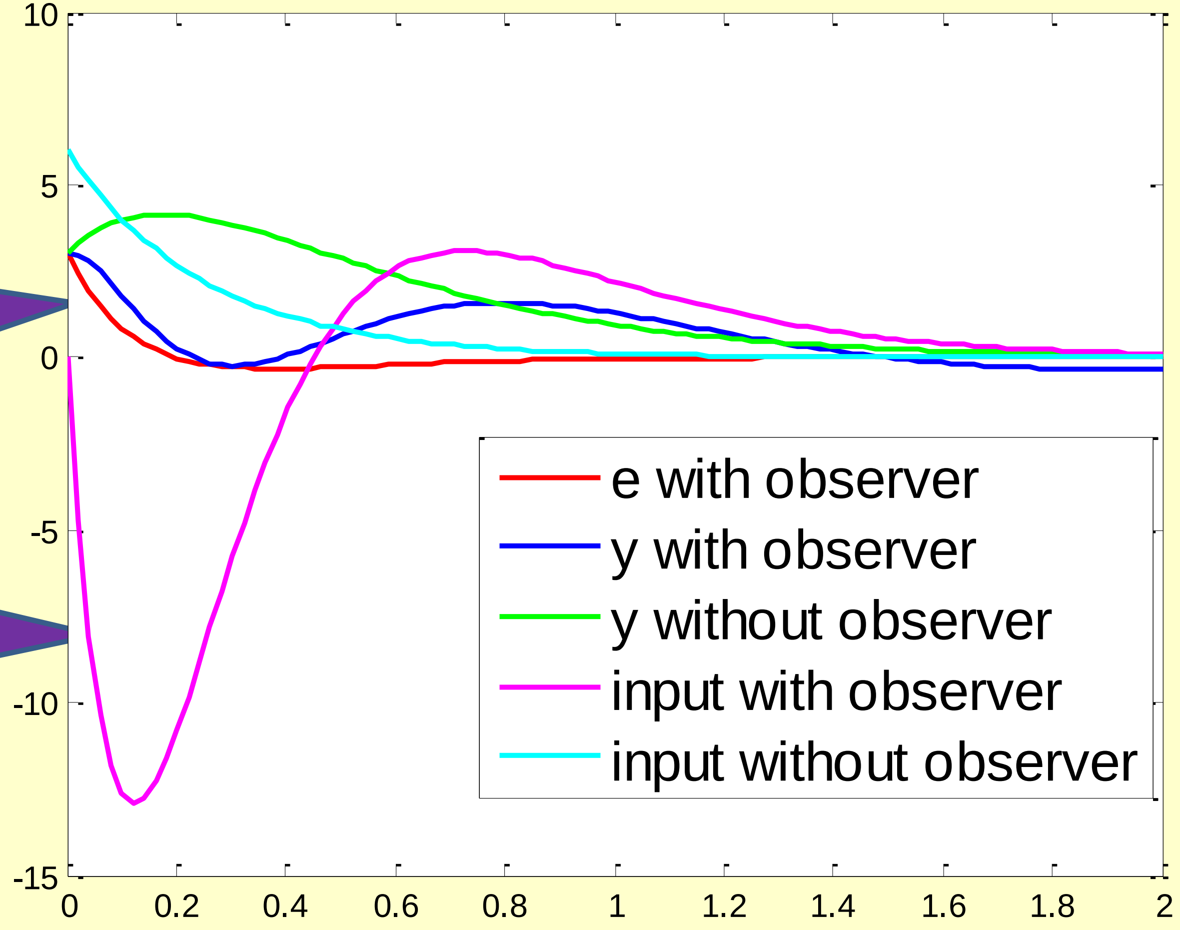
Transient input very large



Make observer poles 2x faster than system poles so that state errors converge moderately quickly

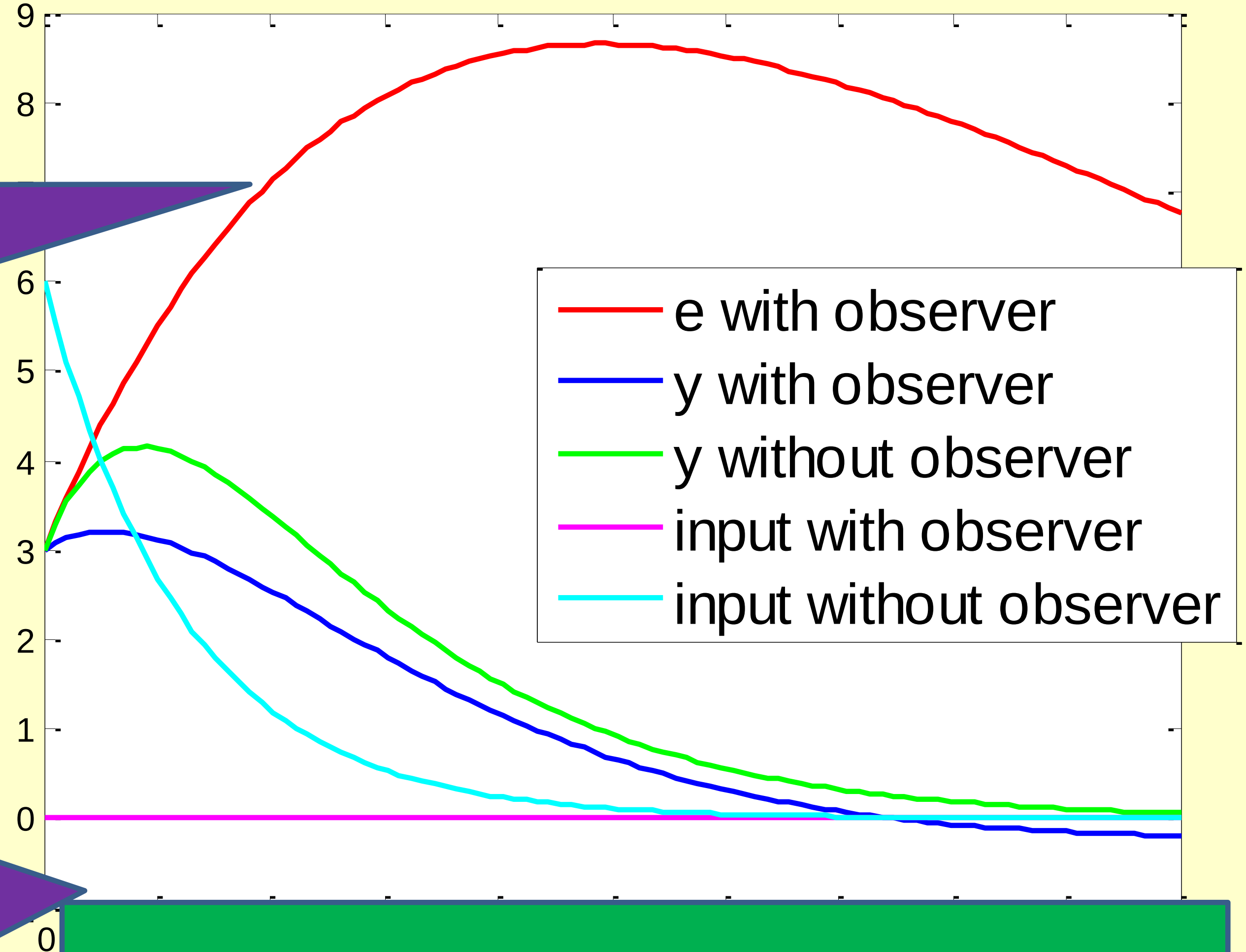
Fast error convergence

Transient input still large



Make observer poles slower than system poles.

Poor error convergence



Transient input small as observer state slow to move.

Behaviour in effect open-loop!

Conclusion

Introduced the separation principle used for analysing system stability with state feedback and a state observer.

1. Shown that one can independently design the state feedback and observer gain to give specified poles and these poles are inherited by the overall closed-loop system, in the absence of parameter uncertainty.
2. Nevertheless, stability and performance are different things and it is less straightforward to determine the impact of observer pole selection on overall closed-loop behaviour.
3. Consideration of these issues and others is beyond the current remit of this video series.

Anthony Rossiter
Department of Automatic Control and Systems Engineering
University of Sheffield
www.shef.ac.uk/acse